Exercise 2 / page 104

Let us find as an example the probability that the first die lands on 6 given that the sum is 9

P(6| sum of 9)=P(first is 6 and the sum is 9)/P(sum is 9)=P(first is 6 the second is 3) / P(sum is 9)

P(first is 6 ,second is 3)=P(first is 6)*P(second is 3) (independence)=(1/6)*(1/6)=1/36
P(sum is 9)=P((3,6),(4,5),(5,4),(6,3)) =4*P((3,6))=4*(1/6)*(1/6)=4/36

So our probability is (1/36)/(4/36)=1/4

The other probabilities will be :

P(6|sum of 7)=P((6,1))/(1/6)=1/6
P(6 |sum of 8)=1/5
P(6|sum of 10)=1/3
P(6|sum of 11)=1/2
P(6|sum of 12)=1

Exercise 13 / page 104

P(D)=.36 , P(C | D)=.22 ,P(C)=.30

a) P(DC)=P(D)*P(C | D)=.0792
b) P(D | C)=P(DC)/P(C)=.0792/.3=.264

Exercise 16 / page 105

P(F)=.52 , P(C)=.05 , P(FC)=.02

a) P(F | C)=P(FC)/P(C)=.02/.05=.40
b) P(C | F)=P(FC)/P(F)=.02/.52=.038

Exercise 24 / page 106

Let A denote the event that the next card is the ace of spades and let B be the event that it is the two of clubs

a) P(A)=P(next card is an ace)P(A | next card is an ace) =(3/32)*(1/4)=3/128=.023

b) Let C be the event that the two of clubs appeared among the first 20 cards

P(B) = P(B | C)P(C)+P(B | C')P(C')=0*(19/48)+(1/32)*(29/48)=29/1536=.019

Exercise 38 / page 108

P(A)=.50 , P(B)=.30 , P(C)=. 20
P(F | A)=.02 , P(F| B)=.03 , P(F | C)=.05

We want to find
P(A | F)= P(AF)/P(F)=P(F | A)P(A)/(P(F | A)P(A)+P(F | B)P(B)+P(F | C)P(C))=

= (.02)(.5)/[(.02)(.5)+(.03)*(.3)+(.05)*(.2)}=10/29=.345

Exercise 49 / page 109

Let W and F be the events that component 1 works and that the system functions

Then P(W)=.5

We need P(W | F)=P(WF)/P(F)=P(W)/(1-P(F'))=.5/(1-.5^n)

This is because P(F')=P(system does not work)=P(no component works)=P(W')^n (the components are independent of each other) =(1-.5)^n=.5^n

Theoretical exercises :

Exercise 1 / page 115

P(A)>0 .We want to show that P(AB | A)>=P(AB | A or B)

But P(AB | A)= P(ABA)/P(A)=P(AB)/P(A) (*)
P(AB | A or B)=P(AB and (A or B))/P(A or B)=P(AB)/P(A or B) (**) because AB is included in (A or B)

Now ,we have to compare (*) and (**) and see which one is larger .But both have the same numerators and as P(A or B) is always greater or equal to P(A) then clearly (**) will be smaller ,which concludes the proof

Exercise 6 / page 116

E1,E2,.....,En are independent

P(E1 or E2 ..or En)=1-P((E1 or E2 or...or En)') = 1- P((E1)' and (E2)' and (E3)' and...

and (En)') =1- P((E1)')*P((E2)')*...*P((En)')=1-(1-P(E1))*(1-P(E2))*..*(1-P(En))

We used Morgan's law for the second equality sign and the independence of the sets for the third .