Stat 100A, Fall 2002, Homework Solutions

Exercise 20/page 173: a)

P(X>0)=P(win first bet}+P(lose,win,win)=18/38 + (20/38)(18/38)^2 =.5918

b) No,because if the gambler wins then he or she wins $1 .However a loss would be either $1 or $3

c) X can be 1(lose,win,win or win) or –1(lose,win,lose or lose,lose,win) or -3 (lose,lose,lose) so

E(X)=1[18/38+(20/38)(18/38)^2]-[(20/38)(20/38)(18/38)2]-3(20/38)^3=
=-.108

Exercise 29/page 175

If check1,then 2 :expected cost=C1+(1-p)C2+pR1+(1-p)R2
If check 2,then 1:Expected cost=C2+pC1+pR1+(1-p)R2 so the first is best if

C1+(1-p)*C2<=C2+p*C1 or C1<=[p/(1-p)]*C2

Exercise 38 / page 176

a) E[(2+X)^2]=Var(2+X)+(E[2+X])^2=Var(X)+9=14
b) Var(4+3*X)=9*Var(X)=45

Exercise 40 / page 176

X~ Bi(5,1/3)

P(X>=4)=C(5,4)*(1/3)^4*(2/3)+(1/3)^5=11/243=.045

Exercise 43 / page 176

X=# wrong digits ~ Bi(5,.2)

$P(X>=3)=C(5,3)(.2)^3(.8)^2+C(5,4)(.2)^4(.8)+(.2)^5=.05792$

Exercise 57/ page 177

$X\sim Po(3)$

a) $P(X>=3)=1-P(X<3)=1-exp(-3)-3*exp(-3)-exp(-3)*3^2/2=$
$=1-exp(-3)*(17/2)=.577$

c) $P(X.>=3|X>=1)=P(X>=3,X>=1)/P(X>=1)=P(X>=3)/P(X>=1)$

$P(X>=1)=1-P(X=0)=1-exp(-3)=1-.049=.951$

So $P(X >= 3 \mid X>=1)=.577/.951=.607$

19. $$E[X^n] = \sum_{i=0}^{\infty} i^n e^{-\lambda} \lambda^i / i!$$

$$= \sum_{i=1}^{\infty} i^n e^{-\lambda} \lambda^i / i!$$

$$= \sum_{i=1}^{\infty} i^{n-1} e^{-\lambda} \lambda^i / (i-1)!$$

$$= \sum_{j=0}^{\infty} (j+1)^{n-1} e^{-\lambda} \lambda^{j+1} / j!$$

$$= \lambda \sum_{j=0}^{\infty} (j+1)^{n-1} e^{-\lambda} \lambda^{j} / j!$$

$$= \lambda E[(X+1)^{n-1}]$$

Hence $[X^3] = \lambda E(X+1)^2]$

$$= \lambda \sum_{i=0}^{\infty} (i+1)^2 e^{-\lambda} \lambda^i / i!$$

$$= \lambda \left[\sum_{i=0}^{\infty} i^2 e^{-\lambda} \lambda^i / i! + 2 \sum_{i=0}^{\infty} i e^{-\lambda} \lambda^i / i! + \sum_{i=0}^{\infty} e^{-\lambda} \lambda^i / i! \right]$$

$$= \lambda[E[X^2] + 2E[X] + 1)$$

$$= \lambda(\mathrm{Var}(X) = E^2[X] + 2E[X] + 1)$$

$$= \lambda(\lambda + \lambda^2 + 2\lambda + 1) = \lambda(\lambda^2 + 3\lambda + 1)$$