

Chapter 6, Prob. 1, p. 290

Two fair dice are rolled. Find the joint probability mass function of X & Y, when:

1. X is the largest number that turns up and Y is the total sum;
2. X is the value of die one and Y is the largest of the two values;
3. X is the smallest and Y is the largest value.

Chapter 6, Prob. 8, p. 290

The joint PDF of X and Y is given by

$$f(x, y) = c(y^2 - x^2)e^{-y}, -y \leq x \leq y; 0 < y < \infty.$$

1. Find the normalizing constant c;
2. Find the marginal densities of X and Y;
3. Find E(X).

Chapter 6, Prob. 10, p. 290

The joint PDF of X and Y is given by $f(x, y) = e^{-(x+y)}, 0 \leq x, y < \infty$. Find $P\{X < Y\}$ and $P\{X < a\}$.

Chapter 6, (Theoretical) Prob. 5, p. 296

If X and Y are independent continuous random variables, express the density function of $Z=X/Y$ and $W=X*Y$, in terms of the densities of X and Y. Evaluate these expressions in the special case when both X and Y are exponential variables.

Chapter 7, Prob. 3, p. 380

If X and Y are independent Uniform(0,1) continuous random variables, show that

$$E\left(|X - Y|^\alpha\right) = \frac{2}{(\alpha + 1)(\alpha + 2)}, \text{ for } \alpha > 0.$$

Chapter 7, Prob. 5, p. 380

A county hospital is located in the center of a square of sides 3 miles wide. In case of an emergency call from within this range the hospital sends an ambulance. Suppose the road network is rectangular, so travel distance from the hospital (with coordinates (0,0)) to any location (x,y) in the square is just $|x|+|y|$. Suppose accidents happen uniformly distributed in the square. Find the expected travel distance for an ambulance for an average call.

Chapter 7, (Theoretical) Prob. 4, p. 390

Let X be a continuous random variable with finite expectation μ_x and variance σ_x^2 . Let $g(\cdot)$ be a twice differentiable function. Show that $E(g(X)) \approx g(\mu) + \frac{g''(\mu)}{2} \sigma^2$. Hint: Expand $g(\cdot)$ using Taylor expansion about $X=\mu$, using only the first 3 terms, ignoring the remainder.