Stat 13 Homework 6 Solutions

Note: Numerical answer keys are in **bold**.

6.1 Let X be the random variable of the daily gold price in a particular month

T1=4X1 T2=X1+X2+X4+X4

a. T1 mean: $E(T1)=E(4X1)=4E(X1)= 4\times1250=5000$
   T1 standard deviation: $SD(T1)=SD(4X1)=4SD(X1)=4\times28=112$
   T1~N(5000, 112)
b. T2 mean: $E(T2)=E(X1+X2+X4+X4)=E(X1)+E(X2)+E(X3)+E(X4)$
   $=1250\times4=5000$
   T2 standard deviation: $SD(T2)=SD(X1+X2+X4+X4)$
   $=\sqrt{}SD(X1)^2+SD(X2)^2+SD(X3)^2+SD(X4)^2$
   $=\sqrt{}4\times28^2=2\times28=56$
   T2~N(5000, 56)
c. The two models have the same mean, but differ in their standard deviations: SD(T1)-SD(T2)=112-56=56
d. $P(T1>5100)=P(z>(5100-5000)/112)=P(z>0.893)=1-0.813=$**0.187**
e. $P(T2>5100)=P(z>(5100-5000)/56)=P(z>1.786)=1-0.962=$**0.038**
f. The second plan appears better because the total amount is less variable as a result of the smaller standard deviation, and thus less likely to run over the budget.

6.2 It's helpful to draw this table of sums of two dice values.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

In case of rolling a single die,

$m_{\mathbf{Y1}}=E(Y_i)=\sum (Y_i\times P(Y_i))=2\times1/64+3\times2/64+4\times3/64+\ldots+16\times1/64=(2+6+12+\ldots+16)/64$

$=524/64=576/64=9$

$s_{\mathbf{Y1}}=SD(Y_i)= \sqrt{}[(2-9)^2+(3-9)^2+\ldots+(16-9)^2]/(64-1)=3.266$

So when rolling fice dice,

$m_{\mathbf{Y}}=E(5Y_i)=5E(Y_i)^2=5\times9=$**45**

$s_{\mathbf{Y}}=\sqrt{}5s_{\mathbf{Y1}}^2=\sqrt{}5\times3.266=$**7.3**

Based on the Central Limit Theorem, $\mathbf{Y}^-$ would be approximately normally distributed. The mean is **45**, and the standard deviation is **7.3**.

6.3 Let x1 be the normal controls group, and x2 be the Mnemonic group.

a. Mean of x1: x1(*bar*)=**9.6316**
   Standard deviation of x1: s1=**3.3368**
   Mean of x2: x2(*bar*)=**14.1**
   Standard deviation of x2: s2=**2.4688**

b. x1(*bar*)-x2(*bar*)±t√$s1^2$/n1+$s2^2$/n2
   =9.6316-14.1±2.101√3.3368^2/19+2.4688^2/20
   =-4.4684±2.101×0.9438
   =-4.4684±1.9829
   =**[-6.4513,-2.4855]**
   If we were to draw repeated samples for the two groups, in 95% of the cases we would expect to capture the difference of their true population means in this range.

c. It would require a sample size of 39×4=**156** to yield a CI of half the size found above.

d. Yes, it should. If we were to draw repeated samples for a normal controls group of 76 participants and an Mnemonic group of 80 participants, in 95% of the cases we would expect to capture the difference of their true population means in this range.