Solution to Homework 5

Problem 1

(1) The variation of the weights is the difference between the coins, which comes from the worn equipment and other variation during manufacturing process.

(2) The procedures in STATA are as follows:

- load the data by

```
.insheet using
"http://www.stat.ucla.edu/%7Edinov/courses_students.dir/data.dir/pennies.dat"
```

- Calculate mean and standard deviation of the weights of the 100 coins by

```
.summarize
```

```
    Variable |     Obs        Mean    Std. Dev.       Min        Max
-------------+-----------------------------------------------------
      weight |     100    3.108306    .0426555     2.9847     3.2014
```

- Construct confidence interval

```
.ci weight, level(90)
```

```
    Variable |     Obs        Mean    Std. Err.       [90% Conf. Interval]
-------------+-------------------------------------------------------------
      weight |     100     3.108306    .0042656        3.1012   3.1154
```

By hand, the 90% confidence interval for the population mean is
3.108306 +/- .0426555/10 * 1.66039, where t (0.95, 99) = 1.66039,
which is, [3.1012 3.1154]

Population of interest: newly minded U.S. pennies.

Interpretation of the CI: 90% of confidence intervals constructed by repeated sampling of 100 newly minded US pennies will cover the true population mean weight in interval [3.1012 3.1154].

(3) 90% prediction interval for the weight of another coin is

3.108306 +/- .0426555/(1+1/100)^0.5 * 1.66039
i.e., [3.0378 3.1788]

Interpretation of the PI: I'm 90% confident that the next coin has weight between 3.0378 and 3.1788.

Problem 2

The 95% confidence interval of the difference on percentage of people whose conditions were improved between treatment group and control group is

(100/150 – 60/100) +/- 1.96*((100/150)*(1-100/150)/150 + (60/100)*(1-60/100)/100)^0.5
i.e. [-0.0554 0.1888]

Since this confidence interval contains zero, there is not enough evidence from this experiment to support the manufacturer's claim that the drug is an effective pain reliever for arthritis patients.

Problem 3

For 0<P<1, let

$f(P) = P(1-P)$

$FOC \qquad \frac{\partial f}{\partial P} = 1 - 2P$

$\Rightarrow \hat{P} = 1/2$

It is a maximum since $\frac{\partial^2 f}{\partial P^2} = -2 < 0$.

Therefore, $f(P) = P(1-P) \leq f(\hat{P} = 1/2) = 1/2(1-1/2) = 1/4$

Y~Binomail (n, P)
Var (Y) = nP(1-P) <= n*(1/4) = n/4