Solution to Homework 6

Problem 6_1

a)   The data from the experiment is summarized as below.

Environment:	1	2	3	4	5	6	7	8	9	Mean	SD
Brand **A**:	23	17	28	48	10	36	15	22	94	32.56	25.759
Brand **B**:	36	22	25	60	16	34	28	22	104	38.56	27.6624
Difference:	-13	-5	3	-12	-6	2	-13	0	-10	-6	6.442

b)   Two-sided t-test treating the two groups from independent populations:

$$H_0 : \mu_1 = \mu_2$$
$$H_a : \mu_1 \neq \mu_2$$
$$t* = \frac{32.56 - 38.56}{\sqrt{\dfrac{25.759^2}{9} + \dfrac{27.6624^2}{9}}} = -0.4762$$
$$P - value = 2P(t < -0.4762) > 0.2$$

This suggests that the differences between the two brands are not statistically significant. The null hypothesis can't be rejected.

Two-sided t-test treating the two groups as paired:

$$H_0 : \mu_1 = \mu_2$$
$$H_a : \mu_1 \neq \mu_2$$
$$t* = \frac{32.56 - 38.56}{\sqrt{\dfrac{6.442^2}{9}}} = -2.794$$
$$P - value = 2P(t_8 > -2.794) \subset 2*[0.01, 0.025]$$

Since the P-value is less than significant level 0.05, there is strong evidence that the two brands are different.

c)   The paired test is more justified. According to the design of the experiment, the 9 individual observations of the two brands are tested under the same environments, so they are paired samples.

Problem 6_2

The hypothesis is on the effectiveness of Sedane-D in lower chance to cause headache as side effect.

$$H_0 : p_1 = p_2$$
$$H_a : p_1 < p_2$$

$$z* = \frac{65/374 - 43/193}{\sqrt{\dfrac{65/374(1 - 65/374)}{374} + \dfrac{43/193(1 - 43/193)}{193}}} = -1.3691$$

$$P - value = P(z < -2.794) = 0.0853$$

This suggests weak evidence against the null hypothesis.

Problem 6_3

One-sided test on the mean breaking strength:

$$H_0 : \mu = 42300$$
$$H_a : \mu < 42300$$

$$t* = \frac{42166 - 42300}{\dfrac{259.41}{2}} = -1.1551$$

$$P - value = P(t_4 < -1.1551) > 0.1$$

It means that the mean breaking strength of the new process is not significantly lower than the requirement.