UCLA STAT 110A - Appplied Statistics, Review

Sampling Distributions of Estimates, CLT

1. A random sample of size n is drawn from a population with mean, μ, and standard deviation, σ. Let $\overline{X}$ be the sample mean.

(a) What is the:

(i) mean of $\overline{X}$?

(ii) standard deviation of $\overline{X}$?

(b) If we are sampling from a Normal distribution then $\overline{X}$ is **exactly / approximately** (circle **one**) Normally distributed.

(c) **(i)** If we are sampling from a non-Normal distribution then for large samples (ie, n is large) $\overline{X}$ is **exactly / approximately** (circle **one**) Normally distributed.

(ii) The result in **(i)** is called the

2. A random sample of size n is drawn from a population in which a proportion p has a characteristic of interest. Let $\hat{P}$ be the sample proportion.

(a) What is the:

(i) mean of $\hat{P}$?

(ii) standard deviation of $\hat{P}$?

(b) For large samples $\hat{P}$ is
exactly / approximately (circle **one**) Normally distributed.

3. A _____ is a numerical characteristic of a population.

4. An _____ is a _____

quantity calculated from data in order to estimate an unknown _____.

5. Suppose that $X_1, X_2, \ldots, X_{16}$ is a random sample from a Normal distribution with mean of 50 and a standard deviation of 10. Then the distribution of the sample mean $\overline{X} = \frac{X_1 + X_2 + \ldots + X_{16}}{16}$ has mean, $\mu_{\overline{X}}$, and standard deviation, $\sigma_{\overline{X}}$, given by:

(1) $\mu_{\overline{X}} = 50, \quad \sigma_{\overline{X}} = 6.25$

(2) $\mu_{\overline{X}} = 50, \quad \sigma_{\overline{X}} = 0.625$

(3) $\mu_{\overline{X}} = 800, \quad \sigma_{\overline{X}} = 10$

(4) $\mu_{\overline{X}} = 50, \quad \sigma_{\overline{X}} = 2.5$

(5) cannot be determined because $n = 16$ is too small for the central limit effect to take effect.

6. The distribution of all long-distance telephone calls is approximately Normally distributed with a mean of 280 seconds and a standard deviation of 60 seconds. A random sample of sixteen calls is chosen from telephone company records. Let $\overline{X}$ be the sample mean of sixteen such calls.

(a) Describe the distribution of $\overline{X}$.

(b) Calculate the probability that the sample mean exceeds 240 seconds. Use the output below to help you.

```
Cumulative Distribution Function

Normal with mean = 280.000 and standard deviation = 60.0000

         x     P( X <= x)
  240.0000        0.2525

Cumulative Distribution Function

Normal with mean = 280.000 and standard deviation = 15.0000

         x     P( X <= x)
  240.0000        0.0038

Cumulative Distribution Function

Normal with mean = 280.000 and standard deviation = 3.75000

         x     P( X <= x)
  240.0000        0.0000
```

7. The fuel consumption, in litres per 100 kilometres, of all cars of a particular model has mean of 7.15 and a standard deviation of 1.2. A random sample of these cars is taken.

(a) Calculate the mean and standard deviation of the sample if:

(i) one observation is taken.

(ii) four observations are taken.

(iii) sixteen observations are taken.

(b) In what way do your answers in (a) differ? Why?

8. About 65% of all university students belong to the student loan scheme. Consider a random sample of 50 students. Let $\hat{P}$ be the proportion of these 50 students who belong to the student loan scheme.

(a) In words, describe *p*.

(b) State the distribution of $\hat{P}$.

(c) What is the probability that the sample proportion is more than 70%? Use the output below to help you.

(d) What is the probability that the sample proportion is between 0.45 and 0.55? Use the output below to help you.

Cumulative Distribution Function

Normal with mean = 0.650000 and standard deviation = 0.0674537

x	P(X <= x)
0.4000	0.0001
0.4500	0.0015
0.5000	0.0131
0.5500	0.0691
0.6000	0.2293
0.6500	0.5000
0.7000	0.7707
0.7500	0.9309

9. The owner of a large fleet of courier vans is trying to estimate her costs for next year's operations. Fuel purchases are a major cost. A random sample of 8 vans yields the following fuel consumption data (in km/L):

10.3 9.7 10.8 12.0 13.4 7.5 8.2 9.1

Assume that the distribution of fuel consumption of the vans is approximately Normal.

(a) Calculate the sample mean and the sample standard deviation.

(b) Construct a two-standard-error interval for the mean fuel consumption of all of her vans.

(c) Without doing any calculations specify what happens to the width of the two-standard error interval in the following cases:

(i) the sample standard deviation increases.

(ii) the sample mean decreases.

(iii) the sample size increases.

10. A large department store wants to estimate the proportion of their customers who have a charge card for the store. They take a random sample of 120 shoppers. They find that 36 of these shoppers have a charge card for the store. Construct a two-standard-error interval for the proportion of all of their customers who have a charge card for the store.

11. Which **one** of the following statements is **true**?

(1) In a poll, all estimates of population proportions, including estimates for subgroups of the population, will have the same standard error.

(2) If *X* is Normal, then the Student's *t*-distribution is used instead of the standard Normal distribution for the distribution of $\frac{\overline{X} - \mu}{\sigma_X / \sqrt{n}}$ when the population standard deviation is replaced by the sample standard deviation.

(3) The Central Limit Effect can only be detected for sample sizes that are greater than 30.

(4) When sampling, taking a large sample guarantees an accurate estimate of the parameter of interest.

(5) For small samples, the shape of the distribution of the sample mean, $\overline{X}$, is always Normal regardless of the shape of the distribution of the random variable *X*.

UCLA Stat 110A - Applied Statistics Review

Confidence Intervals

Section A: Confidence intervals for a mean, proportion and difference between means

1. An exam had a possible total of 64 points.
A random sample of 30 scores was selected from all of the exams. The data collected and its summary is as follows:

46	32	24	20	51	33	35	43	26	29	59	41	30	35	49
53	32	50	52	23	25	53	51	34	26	29	40	38	45	42

sample size	sample mean	sample standard deviation
30	38.20	10.85

You will use this sample to construct a 95% confidence interval for the mean.

(a) State the parameter θ (using a symbol and in words).

(b) State the estimate $\hat{\theta}$ (using a symbol, in words and as a number).

(c) Calculate se($\hat{\theta}$).

(d) State the value of *df*.

(e) Use the table for the Student's *t*-distribution to write down the value of the *t*-multiplier.

(f) Calculate the 95% confidence interval for the mean.

(g) Interpret the confidence interval.

(h) Does the confidence interval contain the true mean? Discuss briefly.

2. Tuberculosis (TB) is known to be a highly contagious disease. In 1995 a study was carried out on a random sample of 1074 Spanish prisoners. The study investigated factors that might be associated with the tuberculosis infection. Some of the results follow.

	Prisoners with tuberculosis	Total number of prisoners
Male	556	984
Female	36	90

You will use this sample to construct a 95% confidence interval for the proportion of female prisoners who had tuberculosis.

(a) State the parameter θ (using a symbol and in words).

(b) State the estimate $\hat{\theta}$ (using a symbol, in words and as a number).

(c) Calculate se($\hat{\theta}$).

(d) Use the table for the Student's *t*-distribution to write down the value of the *z*-multiplier.

(e) Calculate the 95% confidence interval for the proportion of female prisoners who had tuberculosis.

(f) Interpret the confidence interval.

(g) Does the confidence interval contain the true proportion? Discuss briefly.

3. .

Banford et al. [1982] noted that thiol concentrations within human blood cells are seldom determined in clinical studies, in spite of the fact that they are believed to play a key role in many vital processes. They reported a new reliable method for measuring thiol concentration and demonstrated that, in one disease at least (rheumatoid arthritis), the change in thiol status in the lysate from packed blood cells is substantial. There were two groups of volunteers, the first group being "normal" and the second suffering from rheumatoid arthritis. We shall treat the two groups as random samples from the normal and rheumatoid populations respectively (for the area in which the study was undertaken) and will estimate μ_R - μ_N, the difference in true mean thiol levels between the rheumatoid and normal populations.

Computer Output

```
Two sample T for Rheumatoid vs Normal

          N      Mean     StDev   SE Mean
Rheumato  6     3.465     0.440      0.18
Normal    7    1.9214    0.0756     0.029

95% CI for mu Rheumato - mu Normal: ( 1.08,  2.012)
T-Test mu Rheumato = mu Normal (vs not =): T = 8.48  P = 0.0004  DF = 5
```

(a) Interpret the confidence interval.

(b) Does the confidence interval contain the difference in true mean thiol levels between the rheumatoid and normal populations? Discuss briefly.

4. .

Consider constructing a confidence interval for the mean of a population. Which of the following would have an effect on the width of the confidence interval?

I: The size of the sample used to construct the interval.

II: The confidence level for the interval.

III: The amount of variability in the population.

---

(1) I only.

(2) I and II only.

(3) I and III only.

(4) I, II, and III.

(5) II and III only.

5. .

A 95% confidence interval for the difference between the mean haemoglobin levels of people with Type III and people with Type II sickle cell disease, $\mu_{\text{Type III}}$ - $\mu_{\text{Type II}}$, is [-0.80, 2.39]. A **correct** interpretation of this interval would be:

(1) Since zero is in the interval, there is a difference between the average haemoglobin levels for people with Type II sickle cell disease and people with Type III sickle cell disease.

(2) We estimate, with 95% confidence, the average haemoglobin level for people with Type III sickle cell disease to be somewhere between 0.80g/dL lower and 2.39g/dL higher than the average haemoglobin level for people with Type II sickle cell disease.

(3) We estimate, with 95% confidence, the average haemoglobin level for people with Type II sickle cell disease to be somewhere between 0.80g/dL lower and 2.39g/dL higher than the average haemoglobin level for people with Type III sickle cell disease.

(4) On average, people with Type II sickle cell disease have a lower haemoglobin level than people with Type III sickle cell disease.

(5) Since zero is in the interval, there is no difference between the average haemoglobin levels for people with Type II sickle cell disease and people with Type III sickle cell disease.

Questions 6 and **7** refer to the following information.

.

In 1990 *CNN/Time* sought information on how young American adults viewed their parents' marriage. In a telephone poll, one of the questions they asked of six hundred and two (602) 18-29 year old Americans was "Would you like to have a marriage like the one your parents have?" Forty-four percent (44%) responded "Yes".

6. *CNN/Time* were interested in determining what proportion of the 18-29 year old American population would answer "Yes" to this question. Which **one** of the following statements is **false**?

(1) The value of the parameter of interest is an unknown quantity.

(2) In this context, 0.44 is an estimate for the parameter of interest.

(3) The parameter of interest depends on the sample and hence is a random quantity.

(4) A confidence interval for the parameter of interest will give a range of possible values for this parameter.

(5) The parameter of interest is the proportion of 18-29 year old Americans who would have answered "Yes" in 1990.

7. An approximate 95% confidence interval for the proportion of the 18-29 year old American population who would have answered "Yes" to this question in 1990 is [0.400, 0.480]. If two thousand four hundred (2400) 18-29 year old Americans had been sampled instead of six hundred and two (602) 18-29 year old Americans, then the new 95% confidence interval would be approximately:

(1) twice as wide.

(2) one-quarter as wide.

(3) half as wide.

(4) four times as wide.

(5) equally as wide.

8. **.**

The *Listener/Heylen* poll from August 6, 1994 reported results on what New Zealanders think about the "Ten Commandments" from a sample of 1000 randomly chosen New Zealanders. A 99% confidence interval for the proportion of New Zealanders who believed that the commandment "I am the Lord your God; worship no god but me" fully applied to them, p_G, is given by (0.282, 0.358). Which **one** of the following statements is **true**?

(1) The interval (0.282, 0.358) will cover the true, but unknown parameter p_G for 99% of samples taken.

(2) Between 28.2 and 35.8 per cent of New Zealanders believe that this commandment fully applies to them 99% of the time.

(3) A 95% confidence interval for p_G would be wider than this interval.

(4) The probability that the interval (0.282, 0.358) covers the sample proportion is 0.99.

(5) The probability that another interval calculated in the same way from a new sample of 1000 New Zealanders covers p_G is 0.99.

9. **.**

The results of a survey of 1146 New Zealanders were published in the 23 March 1992 issue of Time magazine. In response to the question "Is it a good time to buy a major household item?" 585 respondents replied "yes", 332 replied "no" and 229 replied "don't know".

Let p represent the true proportion of New Zealanders who think it is a good time to buy a major household item. Using the results of this survey a 99% confidence interval for p and a 95% confidence interval for p were constructed. A two standard error interval for p was also constructed.

Which **one** of the following statements is **true**?

The 99% confidence interval would:

(1) be completely contained by the corresponding 95% confidence interval for p.

(2) be narrower than the corresponding two standard error interval for p.

(3) be wider if a much larger sample had been taken.

(4) be wider than the corresponding 95% confidence interval for p.

(5) have confidence limits which are twice as far apart as the confidence limits for the corresponding 95% confidence interval for p.

Section B: Confidence interval for a difference in proportions

1. In 1991 a random sample of New Zealand adults were surveyed about their working hours and the number of jobs they had. A similar survey was carried out in 1994.

Identify the sampling situation as:

Situation (a): *Two independent samples*,
Situation (b): *Single sample, several response categories*,
Situation (c): *Single sample, two or more Yes/No items*,

in the following cases.

(a) We want to compare the proportion of females working 1-39 hours in 1994 with the proportion of females working 40 hours or more in 1994.

(b) We want to compare the proportion of males working 40 hours or more in 1991 with the proportion of females working 40 hours or more in 1991.

(c) In the same survey people were also asked if they had 2 or more jobs. We want to compare the proportion of people who had 2 or more jobs in 1994 with the proportion of people who worked 40 hours or more per week in 1994.

(d) We want to compare the proportion of females working 40 hours or more in 1994 with the proportion of females working 40 hours or more in 1991.

Questions 2 to 6 refer to the following information.
Tuberculosis (TB) is known to be a highly contagious disease. In 1995 a study was carried out on a random sample of 1074 Spanish prisoners. The study investigated factors that might be associated with the tuberculosis infection. The results follow.

Variable		Prisoners with tuberculosis	Total number of prisoners
Gender	Male	556	984
	Female	36	90
Race	White	496	886
	Gypsy	74	152
	Other	22	36
Intravenous Drug Users	Yes	361	629
	No	231	445
HIV Positive	Yes	186	294
	No	406	780
Re-imprisonment	Yes	272	456
	No	320	618

2. Identify the sampling situation as:

Situation (a): *Two independent samples*,
Situation (b): *Single sample, several response categories*,
Situation (c): *Single sample, two or more Yes/No items*,

in the following cases:

(a) Of those prisoners who had TB, we want to compare the proportion of white prisoners with the proportion of Gypsy prisoners.

(b) We want to compare the proportion of male prisoners who had TB with the proportion of female prisoners who had TB.

(c) We want to compare the proportion of prisoners who were intravenous drug users with the proportion of prisoners who had been re-imprisoned.

(d) We want to compare the proportion of white prisoners who had TB with the proportion of Gypsy prisoners who had TB.

(e) Of those prisoners who had TB, we want to compare the proportion who were intravenous drug users with the proportion who were HIV-positive.

(f) We want to compare the proportion of Gypsy prisoners with the proportion of prisoners whose race was categorised as "other".

3. The standard error of the difference between the proportion of prisoners who have TB that are intravenous drug users and the proportion of prisoners who have TB that are HIV positive is:

(1) $\sqrt{\dfrac{0.6098(1-0.6098)+0.3142(1-0.3142)}{592}}$

(2) $\sqrt{\dfrac{0.6098+0.3142-(0.6098-0.3142)^2}{592}}$

(3) $\sqrt{\dfrac{0.6098+0.3142+(0.6098-0.3142)^2}{592}}$

(4) $\sqrt{\dfrac{0.6098(1-0.6098)}{629}+\dfrac{0.3142(1-03142)}{294}}$

(5) $\sqrt{\dfrac{0.6098^2+0.3142^2}{592}}$

4. Construct a 95% confidence interval for the difference between the proportion of White prisoners who were infected with TB and the proportion of Gypsy prisoners who were infected with TB. State what your interval tells you in plain English.

(a) State the parameter θ (using symbols and in words).

(b) State the estimate $\hat{\theta}$ (using symbols, in words and as a number).

(c) Calculate se($\hat{\theta}$).

(d) Use the table for the Student's *t*-distribution to write down the value of the *z*-multiplier.

(e) Calculate the confidence interval.

(f) Interpret the confidence interval.

5. Construct a 95% confidence interval for the difference in the proportion of prisoners infected with TB who were white and the proportion of prisoners infected with TB who were Gypsy.

(a) State the parameter θ (using symbols and in words).

(b) State the estimate $\hat{\theta}$ (using symbols, in words and as a number).

(c) Calculate se($\hat{\theta}$).

(d) Use the table for the Student's *t*-distribution to write down the value of the *z*-multiplier.

(e) Calculate the confidence interval.

(f) Interpret the confidence interval.

6. Let p_Y be the proportion of intravenous drug user prisoners who were infected with TB, and p_N be the proportion of non-intravenous drug user prisoners who were infected with TB. The *Excel* worksheet below shows the calculations for a 95% confidence interval based on the data shown on the first page.

Two population proportions

Input data	
X1_sample	361
X2_sample	231
n1_total	629
n2_total	445
p1_ratio	0.573926868
p2_ratio	0.519101124
pdiff	0.054825744
Alpha	0.05
Calculated value	
se	0.03081794
t-multiplier	1.959961082
Confidence Interval	
Lower limit	-0.00557622
Upper limit	0.115227708

(a) Which sampling situation applies here? Briefly explain why.

(b) Interpret the confidence interval.

(c) Is it plausible that p_Y is equal to p_N? Justify your answer.

7. .

A *Time/CNN* poll was based on a telephone survey of 800 adult Hong Kong residents conducted two weeks before the hand over of Hong Kong to China. p_c is the proportion of people in Hong Kong who think "Corruption" is the issue which worries them most, and p_f is the proportion of people in Hong Kong who think "Reduced personal freedoms" is the issue which worries them most.

A 95% confidence interval for $p_c - p_f$ is (0.012, 0.088). Which **one** of the following statements is **false**?

(1) In repeated sampling, we would expect that 95% of the 95% confidence intervals produced contain the true value of $p_c - p_f$.

(2) In light of the data, the interval (0.012, 0.088) contains the most plausible values for $p_c - p_f$.

(3) The true value of $p_c - p_f$ must be in the interval (0.012, 0.088).

(4) At this level of confidence, statements such as "p_c is bigger than p_f by somewhere between 0.012 and 0.088" are true, on average, 19 out of 20 times.

(5) With 95% confidence, the true value of $p_c - p_f$ is 0.05 with a margin of error of ±0.038.

8. .

In a Time Morgan poll (July 1994) 662 voters were interviewed by telephone and asked whether *developing the economy* or *protecting the environment* would be more important in the short term. There were 238 National and 162 Labour supporters in the poll.

Let p_N be the true proportion of National supporters and let p_L be the true proportion of Labour supporters who think that *protecting the environment* is more important in the short term. A 95% confidence interval for the difference between the proportions p_N - p_L is given by [-0.1526, 0.0326]. Which **one** of the following interpretations is **true**?

(1) With a probability of 0.95, the true difference of proportions p_N - p_L lies between –0.1526 and 0.0326.

(2) In repeated sampling the 95% confidence interval [-0.1526, 0.0326] will contain the true difference in proportions in 95% of the samples taken.

(3) In repeated sampling the true proportion p_N will be somewhere between 0.1526 larger and 0.0326 smaller than p_L.

(4) With 95% confidence the true proportion p_N is somewhere between 0.1526 smaller and 0.0326 larger than p_L.

(5) With 95% confidence the true proportion p_N is 0.1852 larger than p_L.

UCLA Stat 110A - Applied Statistics - Review

Hypothesis Testing

Section A:

1. What is the difference between the null hypothesis and the alternative hypothesis?

2. List the three alternative hypotheses that are possible when testing the null hypothesis $H_0: \quad = \theta_0$.

(a) **(b)** **(c)**

3. On what basis do we decide whether to do a one-tailed or a two-tailed test?

θ

4. Use the Formulae Sheet to fill in the formula for the *t*-test statistic:

(a) In words:

(b) In symbols:

5. Fill in the gaps in this description of the *P-value* from a *t*-test:

The *P-value* is the _____ that, if the _____ _____ was true, _____ _____ would produce an estimate that is _____ _____ _____ _____ _____ from the _____ _____ than our _____ _____.

6. What does the *P-value* measure?

7. In the table below, fill in the descriptions used for the given *P-values*:

P-value	**Evidence against H_0**
> 0.12	
≈ 0.10	
≈ 0.05	
≈ 0.01	
< 0.01	

8. What does the *P-value* tell us about the size of the effect?

9. What tool do we use to tell us about the size of an effect?

10. What is a significance test?

11. What does a significant test reveal?

Section B:

1. Tuberculosis (TB) is known to be a highly contagious disease. In 1995 a study was carried out on a random sample of 1074 Spanish prisoners. The study investigated factors that might be associated with the tuberculosis infection. The results follow.

Variable		Prisoners with tuberculosis	Total number of prisoners
Gender	Male	556	984
	Female	36	90
Race	White	496	886
	Gypsy	74	152
	Other	22	36
Intravenous Drug Users	Yes	361	629
	No	231	445
HIV Positive	Yes	186	294
	No	406	780
Re-imprisonment	Yes	272	456
	No	320	618

Is there any evidence to suggest that the race of the prisoner (White or Gypsy) makes any difference to whether they contracted tuberculosis? Carry out a significance test to answer this question and then calculate an appropriate 95% confidence interval.

Let p_W be the proportion of White prisoners infected with TB and p_G be the proportion of Gypsy prisoners infected with TB.

(a) Identify the parameter θ.

(b) State the hypotheses.

(c) Write down the estimate and its value.

(d) Calculate the value of the *t*-test statistic.

(e) Find the *P-value*.

(f) Interpret the *P-value*.

(g) Answer the original question.

(h) Calculate a 95% confidence interval for the parameter.

(i) Interpret the 95% confidence interval.

2. In a poll conducted for TIME and CNN (TIME 17 September 1990, page 51), 1009 residents of New York City were asked "If you could choose where you live, would you live in New York City or move somewhere else?" 595 of the residents said that they would move somewhere else. Could you conclude that this is the opinion of a majority of residents of New York City?

The information below is a Compuiter output for a significance test and a 95% confidence interval. Use this output to answer the questions below.

Test and Confidence Interval for One Proportion

```
Test of p = 0.5 vs p not = 0.5

Sample      X      N  Sample p        95.0 % CI       Z-Value  P-Value
1         595   1009  0.589693  (0.559342, 0.620044)     5.70    0.000
```

(a) State the parameter used in this analysis.

(b) State the hypotheses used in this *t*-test.

(c) Write down the estimate and its value.

(d) Write down the value of the test statistic and the *P-value*.

(e) Answer the original question. (I.e., could you conclude that this is the opinion of a majority of residents of New York City?)

(f) Interpret the confidence interval.

3. A businessperson is interested in buying a coin-operated laundry and has a choice of two different businesses. The businessperson is interested in comparing the average daily revenue of the two laundries, so she collects some data. A simple random sample for 50 days from the records for the past five years of the first laundry and a simple random sample for 30 days from the records for the past three years of the second laundry reveal the following summary statistics:

	Sample size	Sample mean	Sample standard deviation
Laundry 1	50	\$635.40	\$71.90
Laundry 2	30	\$601.60	\$77.70

Computer output

Two Sample T-Test and Confidence Interval

```
Two sample T for Laundry1 vs Laundry2

           N      Mean     StDev   SE Mean
Laundry1  50     635.4      71.9        10
Laundry2  30     601.6      77.7        14

95% CI for mu Laundry1 - mu Laundry2: ( -1,  69)
T-Test mu Laundry1 = mu Laundry2 (vs not =): T = 1.94  P = 0.057  DF = 57
```

```
Stem-and-leaf of Laundry1  N  = 50
Leaf Unit = 10

  1    4 7
  1    4
  2    5 0
  5    5 233
  6    5 5
 10    5 6777
 16    5 889999
 18    6 01
(10)   6 2222233333
 22    6 4444445555
 13    6 67
 11    6 889
  8    7 1
  7    7 23
  5    7 5
  4    7 77
  2    7 99
```

```
Stem-and-leaf of Laundry2  N  = 30
Leaf Unit = 10

  1    4 3
  2    4 6
  9    5 0122344
 15    5 558889
 15    6 113344444
  6    6 77
  4    7 023
  1    7 5
```

(a) State the parameter used in this analysis.

(b) State the hypotheses used in this *t*-test.

(c) Write down the estimate and its value.

(d) Write down the value of the test statistic.

(e) Interpret the test.

(f) Interpret the confidence interval.

(g) Do the stem-and-leaf plots give you any reasons for doubting the validity of the results of this analysis? Briefly explain.

(h) If this analysis were done by hand the value of *df* would have been 29. Why does the output show that *df* = 57?

Section C:

1. Which **one** of the following statements regarding significance testing is **false**?

(1) A highly significant test result means that the size of the difference between the estimated value of the parameter and the hypothesised value of the parameter is significant in a practical sense.

(2) A *P-value* of less than 0.01 is often referred to as a highly significant test result.

(3) A nonsignificant test result does not necessarily mean H_0 is true.

(4) A two-tail test of H_0: $\theta = \theta_0$ is significant at the 5% significance level if and only if θ_0 lies outside a 95% confidence interval for θ.

(5) Testing at the 5% level of significance means that the null hypothesis is rejected whenever a *P-value* smaller than 5% is obtained.

2. Which **one** of the following statements is **false**?

(1) In hypothesis testing, a nonsignificant result implies that H_0 is true.

(2) In hypothesis testing, a two-tail test should be used when the idea for doing the test has been triggered by the data.

(3) In surveys, the nonsampling error is often greater than the sampling error.

(4) Larger sample sizes lead to smaller standard errors.

(5) In hypothesis testing, statistical significance does not necessarily imply practical significance.

3. Which **one** of the following statements regarding significance testing is **false**?

(1) Formal tests can help determine whether effects we see in our data may just be due to sampling variation.

(2) The *P-value* associated with a two-sided alternative hypothesis is obtained by doubling the *P-value* associated with a one-sided alternative hypothesis.

(3) The *P-value* says nothing about the size of an effect.

(4) The data should be carefully examined in order to determine whether the alternative hypothesis needs to be one-sided or two-sided.

(5) The *P-value* describes the strength of evidence against the null hypothesis.

Review UCLA Stat 110A Final Exam

Chapter 7 – Relationships between Quantitative Variables: Regression and Correlation

Section A: The Straight Line Graph

1. The equation of a line is of the form $y = \beta_0 + \beta_1 x$, where β_0 is the y-intercept and β_1 is the slope of the line. Give the values of β_0 and β_1 for the following lines.

(a) $y = 5 + 3x$ **(b)** $y = 10 - 14x$

$\beta_0 =$ $\beta_0 =$

$\beta_1 =$ $\beta_1 =$

2. **(a)** What is the equation of a line that has a slope of 2 and a y-intercept of –3?

(b) By how much does the y-value of this line change when

(i) x is increased by 1?

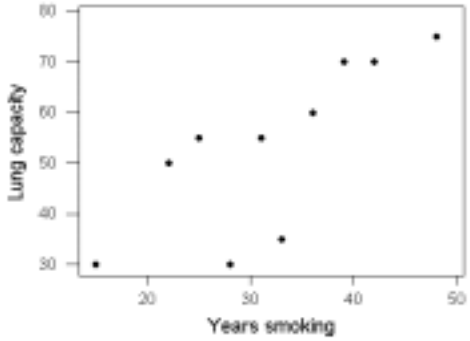
(ii) x is increased by 6?

Section B: Regression

1. Observations on lung capacity, measured on a scale of 0 – 100, and the number of years smoking were obtained from a sample of emphysema patients. One of the uses of the data is to use the number of years smoking to predict lung capacity. The data is shown in the table below. A scatter plot, residual plot, Normal probability plot and *Excel* output are also shown.

Patient	1	2	3	4	5	6	7	8	9	10
Number of years smoking	25	36	22	15	48	39	42	31	28	33
Lung capacity	55	60	50	30	75	70	70	55	30	35

Scatter plot



***Excel* regression output**

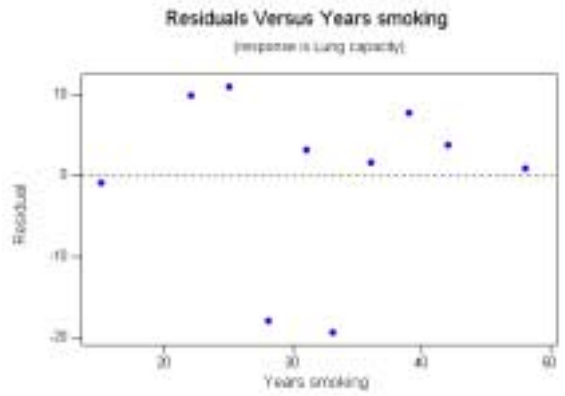
SUMMARY OUTPUT

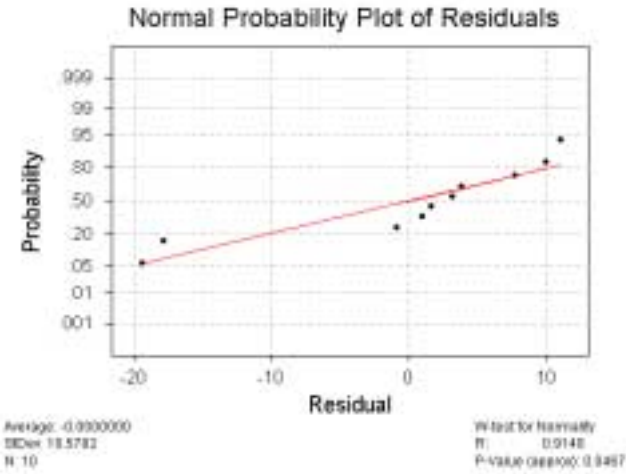
Regression Statistics	
Multiple R	0.773802257
R Square	0.598769933
Adjusted R Square	0.548616175
Standard Error	11.21989008
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1502.912533	1502.912533	11.93868522	0.008627995
Residual	8	1007.087467	125.8859334		
Total	9	2510			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	11.23788345	12.59660909	0.892135603	0.398359228	-17.80996799	40.28573489
Years smoking	1.309157259	0.37889037	3.455240255	0.008627995	0.435433934	2.182880583





(a) Write the equation of the least-squares regression line.

(b) Use the least-squares regression line to predict the lung capacity of an emphysema patient who has been smoking for 30 years.

(c) Patient 1 had smoked for 25 years and had a lung capacity of 55. Calculate the residual (prediction error) for this observation.

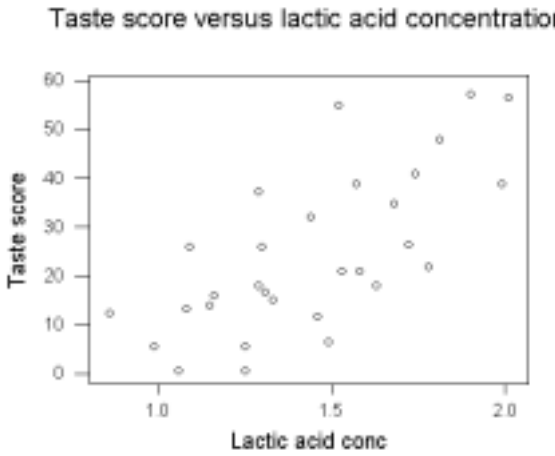
(d) Comment on the appropriateness of using a linear regression model for this data.

(e) Assume that it is appropriate to use a linear regression model for this data. (**Note:** This may not be true.) Carry out a statistical test to see if there is any evidence of an effect of years of smoking on lung capacity. State the hypotheses and interpret the test. If there is evidence of an effect then describe the size of the effect.

(f) **(i)** Find the sample correlation coefficient from the *Excel* output.

(ii) What does *Excel* call it?

2. A study of cheddar cheese from Latrobe Valley investigated the effect on the taste of cheese of various chemical processes that occur during the aging process. One of the aims of the study was to see if the lactic acid concentration could be used to predict the taste score (a subjective measure of taste). Observations were made on 30 randomly selected samples of mature cheddar cheese. A linear regression model is fitted to the data. A scatter plot, residual plot and a Normal probability plot are given below, along with a Normality test and some SYSTAT output.



Regression Analysis

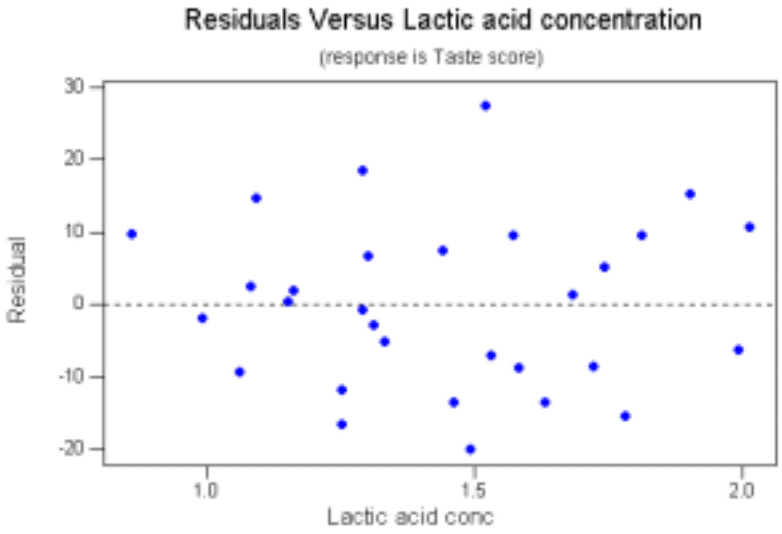
The regression equation is
Taste score = - 29.9 + 37.7 Lactic acid conc

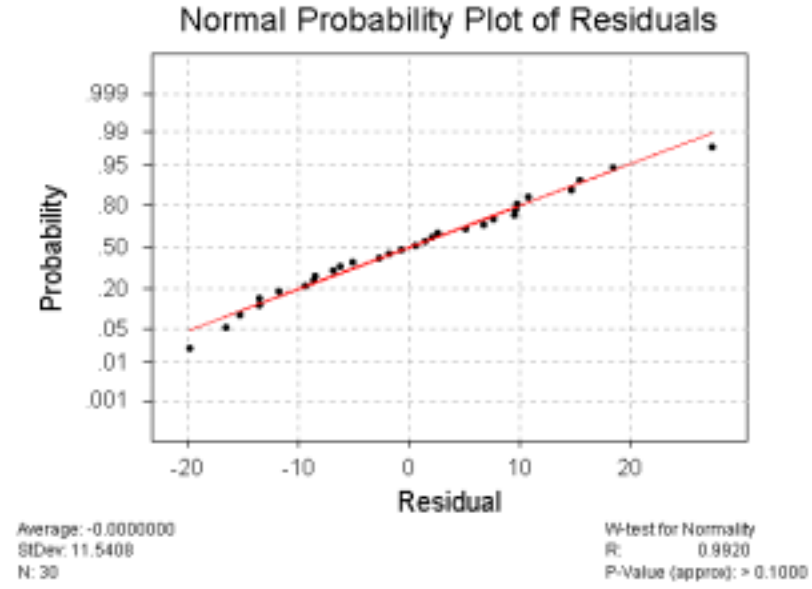
Predictor	Coef	StDev	T	P
Constant	-29.86	10.58	-2.82	0.009
Lactic a	37.720	7.186	5.25	0.000

S = 11.75 R-Sq = 49.6% R-Sq(adj) = 47.8%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	3800.4	3800.4	27.55	0.000
Residual Error	28	3862.5	137.9		
Total	29	7662.9			





(a) One of the observations had a lactic acid concentration of 1.46 and a taste score of 11.6. Calculate the residual for this observation.

(b) Comment on the appropriateness of using a linear regression model for this data.

(c) Assume that it is appropriate to use a linear regression model for this data. (**Note:** This may not be true.) Carry out a statistical test to see if there is any evidence of an effect of lactic acid concentration on taste score. State the hypotheses and interpret the test. If there is evidence of an effect then describe the size of the effect. (Note: For a 95% confidence interval with $df = 28$, the t-multiplier is 2.048.)

(d) The researcher wanted to predict the taste score of a cheddar cheese with a lactic acid concentration of 1.8 and used SYSTAT to produce the following output.

```
Predicted Values

     Fit  StDev Fit          95.0% CI              95.0% PI
   38.04       3.35    (   31.18,    44.90)  (   13.02,    63.05)
```

Use the SYSTAT B output to interpret the following:

(i) The "Fit" value of 38.04.

(ii) The 95% confidence interval.

(iii) The 95% prediction interval.

(e) The fitted least-squares regression line indicates that for each increase of 0.05 in lactic acid concentration we expect that, on average, the taste score will:

(1) increase by approximately 1.9 units.

(2) decrease by approximately 28.0 units.

(3) increase by approximately 37.7 units.

(4) increase by approximately 18.9 units.

(5) decrease by approximately 29.9 units.

(f) The fitted least-squares regression line can be used to predict taste scores for samples of mature cheddar from the Latrobe Valley. Cheese that has a lactic acid concentration of 1.30 has a predicted taste score of:

(1) 24.5

(2) 19.2

(3) 49.0

(4) 78.9

(5) 25.9

Section C.

1. Which **one** of the following statements regarding the sample correlation coefficient, *r*, is **false**?

(1) The value of *r* is an indication of the strength of linear association between the two variables.

(2) In the calculation of the value of *r*, it does not matter which one of the variables is designated as *X* and which one is designated as *Y*.

(3) If the sample correlation coefficient equals 1, then there is a perfect linear association between the two variables for these observations.

(4) The value of *r* must be between 0 and 1 inclusive.

(5) The value of *r* may be near 0 when there is a non-linear relationship between the two variables.

2. In the theory of inference, which **one** of the following is **not** an assumption for the linear regression model?

(1) The mean of the errors is 0 for all *X*-values.

(2) The errors are not independent.

(3) The standard deviation of the errors is the same for all *X*-values.

(4) The relationship between *X* and *Y* variables can be summarised by a straight line.

(5) The distribution of the errors is Normal for all *X*-values.

3. Consider using a scatter plot to investigate the relationship between a response variable *Y* and an explanatory variable *X*. The scatter plot indicates that there is a strong, negative, linear relationship between *X* and *Y* and that there are no outliers in the data. Which **one** of the following statements is **false**?

(1) The trend line explains most of the differences we see between the values of *Y* in the scatter plot.

(2) There are no points that are unusually far from the trend curve.

(3) *Y* changes, on average, by a fixed amount for each unit change in *X*.

(4) The value of *Y* tends to decrease as the value of *X* increases.

(5) If a new scatter plot was produced that only used a limited range of the *X*-values, then the relationship would look stronger.

4. Which **one** of the following statements regarding the sample correlation coefficient, *r*, is **false**?

(1) A value of *r* near 1 does not necessarily mean there is a causal relationship between the two variables.

(2) The value of *r* cannot be less than -1.

(3) In calculating *r*, it is not necessary to define one of the random variables as the response and the other as the explanatory variable.

(4) A negative value of *r* indicates a negative association between the two variables.

(5) A value of *r* equal to 0 indicates that there is no relationship between the two variables.

5. Which **one** of the following statements regarding linear regression and correlation analysis is **false**?

(1) In an analysis of the correlation between two variables, we do not single out either variable to have a special role.

(2) Using regression techniques, we can never determine whether a causal relationship exists between two variables.

(3) An outlier on a scatter plot should be removed if it is found to be an error.

(4) A strong relationship plotted for a limited range of x-values may appear weaker than it actually is.

(5) The least-squares regression technique minimises the sum of the squared prediction errors.

6. Which **one** of the following statements is **not** an assumption of the linear regression model?

(1) The relationship between the X variable and the Y variable is linear.

(2) All random errors are independent.

(3) The X-values are Normally distributed.

(4) The standard deviation of the random errors does not depend on the X-values.

(5) For any X-value, the random errors are Normally distributed (with a mean of 0).

7. Which **one** of the following statements about the sample correlation coefficient, r, between two variables X and Y is **false**?

(1) A value of r close to 1 implies a causal relationship exists between X and Y.

(2) A value of $r = 0$ does not necessarily mean that X and Y are unrelated.

(3) A value of $r = 0$ indicates that no linear relationship exists between X and Y.

(4) A value of $r = 1$ indicates that a perfect positive linear relationship exists between X and Y.

(5) A value of $r = -1$ indicates that a perfect negative linear relationship exists between X and Y.

8. Which **one** of the following statements about linear regression and correlation is **false**?

(1) A regression relationship is of the form:

observation = trend + residual scatter.

(2) In analyses of the correlation type, no variables are singled out to have a special role; all variables are treated symmetrically.

(3) Correlation coefficients provide a better means of detecting a relationship between two continuous variables than a scatter plot.

(4) The fitted trend line is often useful for prediction purposes.

(5) Lines fitted to data using the least-squares method do not allow us to reliably predict the behaviour of Y outside the range of x-values for which we have collected data.

9. Which **one** of the following statements is **false**?

(1) The two main components of a regression relationship are ‘trend’ and ‘scatter’.

(2) The larger the amount of scatter, the smaller the size of the absolute value of the correlation coefficient, r.

(3) A correlation coefficient of $r = 0$ means that there is no linear relationship between the two variables, whereas a negative correlation coefficient indicates an association, the strength of which depends on its absolute value.

(4) A small value of the absolute value of the correlation coefficient, r, indicates a weak linear relationship.

(5) In the interpretation of a correlation coefficient, r, one variable is always treated as the response variable and the other as the explanatory variable.

10. Which **one** of the following statements concerning the analysis of residuals is **false**?

(1) A linear regression model should never be used without first examining the appropriate scatter plot.

(2) Outliers in the values of the explanatory variable can have a big influence on the fitted regression line.

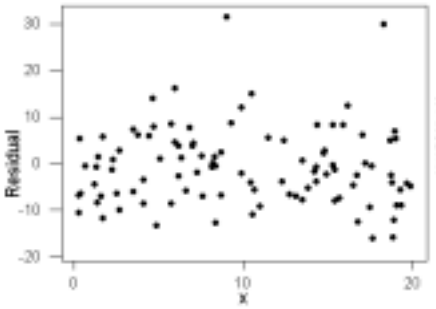
(3) The residuals are computed to be $x_i - \hat{y}_i$.

(4) If the assumption of constant error standard deviation is valid, we would expect to see a patternless horizontal band in a plot of the residuals versus the explanatory variable.

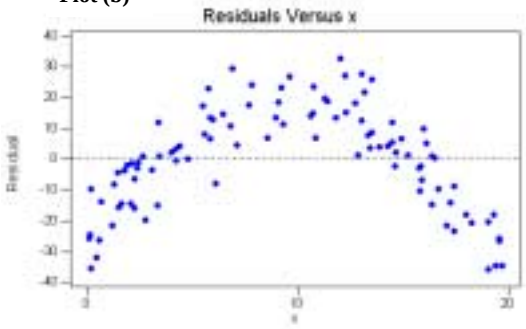
(5) We can investigate the distribution of the errors by looking at a stem-and-leaf plot of a hjistogram of the residuals.

Questions 11 and 12 refer to the following set of residual plots.

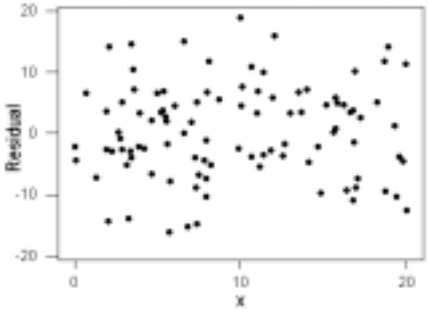
Plot (a)



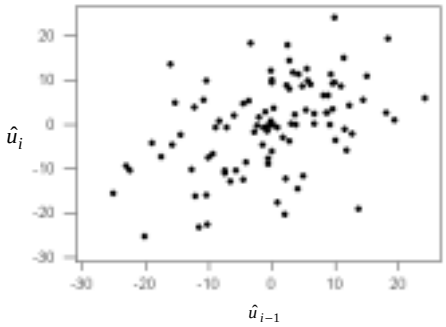
Plot (b)



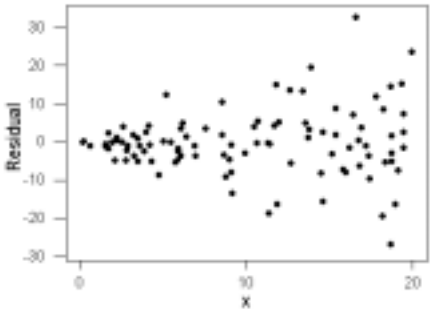
Plot (c)



Plot (d)



Plot (e)



11. Which **one** of the plots does **not** indicate problems with the assumptions underlying the linear regression model?

(1) (a)
(2) (b)
(3) (c)
(4) (d)
(5) (e)

12. Which **one** of the plots indicates that the variability of the error term is **not** independent of x?

(1) (a)
(2) (b)
(3) (c)
(4) (d)
(5) (e)