[Stat13 Homework 6](http://www.stat.ucla.edu/~dinov/courses_students.html)

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(20 points, student scores will be converted to scores out of 100)

Suggested Solutions

Problem 6_1 (6 points)

X ~ N(mean = 1250, sigma = 28)

(1 pt)
(a) T1 = 4X
E(T1) = 4 E(X) = 5000
V(T1) = 16 V(X) = 12544, so SD(T1) = 112
So T1 ~ N(mean = 5000, sigma = 112)

(1 pt)
(b) T2 = X1 + X2 + X3 + X4
E(T2) = E(X1) + E(X2) + E(X3) + E(X4) = 5000
V(T2) = V(X1) + V(X2) + V(X3) + V(X4) = 3136 so SD(T2) = 56

(1 pt)
(c) differ in SD, plan 2 has smaller SD, half of that of plan 1

(1pt)
(d) P(T1 > 5100) = P(Z > (5100-5000)/112) = P(Z > 0.8928) = 0.186

(1pt)
(e) P(T2 > 5100) = P(Z > (5100-5000)/56) = P(Z > 1.7857) = 0.037

(1pt)
(f) Plan 2

Problem 6_2 (6 points)

Let $X_{ij}, i = 1,2,3,4,5, j = 1,2,$ denote the output of i-th die in j-th throw. Then clearly $Y = \sum_i \sum_j X_{ij}$. And we can also see that X_{ij}'s are independent of each other and identically distributed.

$E(X_{11}) = \frac{1}{8}(1+2+3+4+5+6+7+8) = 4.5$ and $\text{var}(X_{11}) = \sum_{i=1}^{8} \frac{1}{8}(i-4.5)^2 = \frac{21}{4}$

hence $\mu_Y = 4.5 * 10 = 45$, (2 points)

and $\sigma_Y = \sqrt{\text{var}(Y)} = \sqrt{10 * \frac{21}{4}} = 7.25$ (2 points)

Approximately, $\bar{Y}$ will follow a Normal distribution by Central Limit theorem.

And $E(\bar{Y}) = E(Y_1) = 45$, (1 point)

$$sd(\overline{Y}) = \frac{1}{\sqrt{n}} sd(Y_1) = 7.25/3 = 2.42$$ (1 point)

<u>Problem 6_3</u> (8 points)

(2 pts)
(a) X1 : Treatment group: mean = 14.1, SD = 2.468
X2 : Control group: mean = 9.63, SD = 3.336

(4 pts)
(b) 95% CI = [(X1-bar – X2-bar) ± t-value * SE(X1-bar – X2-bar)]

SE(X1-bar – X2-bar) = sqrt($SD1^2$/n1 + $Sd2^2$/n2) = sqrt (2.468^2/20 + 3.336^2/19)
= 0.9435

For t-value:
Here, 95% = (1-alpha)*100%, so alpha = 0.05 = 5%
Since we're calculating CI, hence the alpha value in the t-table is 0.05 / 2 = 0.025
DF = min(n1, n2) – 1 = 19 – 1 = 18.
So, $t_{0.025, 18}$ = 2.101

(X1-bar – X2-bar) = 14.1 – 9.63 = 4.47

So 95% CI = (4.47 ± 2.101 * 0.9435) = (2.49, 6.45)

Since this interval is greater than 0 (0 does not lies in the interval), we are 95% confident that there's positive difference between the 2 groups. Mean of treatment group is larger than that of the control group.

(1pt)
(c) To have 0.5 SE(X1-bar – X2-bar) = 0.5 * sqrt($SD1^2$/n1 + $Sd2^2$/n2)
= sqrt(0.25 * ($SD1^2$/n1 + $Sd2^2$/n2))
= sqrt($SD1^2$/ 4*n1 + $Sd2^2$/ 4*n2)

Hence, the new sample size is 4n1 + 4n2 = 4 (n1 + n2) = 4(19+20) = 156

(1pt)
(d) Interval may contain the true difference, but there's a possibility that it doesn't contain it. If we perform such experiment many times, e.g. 100 times, then about 95% of the 100 CI's will contain the true difference.