UCLA STAT 13

Introduction to Statistical Methods for the Life and Health Sciences

- **Instructor**: **Ivo Dinov**,
  Asst. Prof. In Statistics and Neurology
- **Teaching Assistants:**
  Ming Zheng, Sovia Lau, Tianwei Yu
  UCLA Statistics

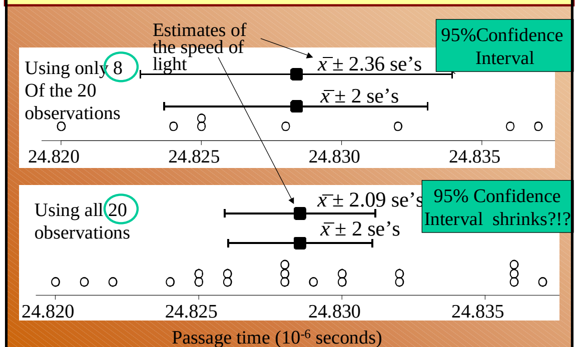
University of California, Los Angeles, Fall 2003

http://www.stat.ucla.edu/~dinov/courses_students.html

Chapter 8: Confidence Intervals

- Introduction
- Means
- Proportions
- Comparing 2 means
- Comparing 2 proportions
- How big should my study be?

20 replicated measurements to estimate the speed of light. Obtained by Simon Newcomb in 1882, by using distant (3.721 km) rotating mirrors.



A 95% confidence interval

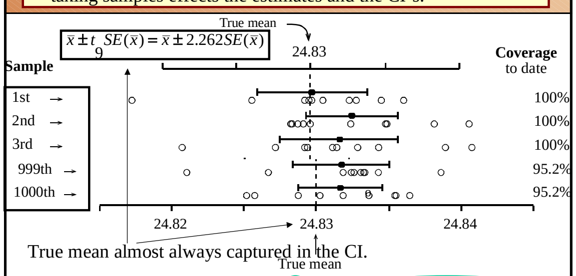
- A type of interval that contains the true value of a parameter for 95% of samples taken is called a ***95% confidence interval*** for that parameter, the ends of the CI are called *confidence limits*.
- (For the situations we deal with) a confidence interval (CI) for the true value of a parameter is given by

  estimate $\pm$ ***t standard errors***

TABLE 8.1.1 Value of the Multiplier, *t*, for a 95% CI

df :	7	8	9	10	11	12	13	14	15	16	17
t :	2.365	2.306	2.262	2.228	2.201	2.179	2.160	2.145	2.131	2.120	2.110
df :	18	19	20	25	30	35	40	45	50	60	∞
t :	2.101	2.093	2.086	2.060	2.042	2.030	2.021	2.014	2.009	2.000	1.960

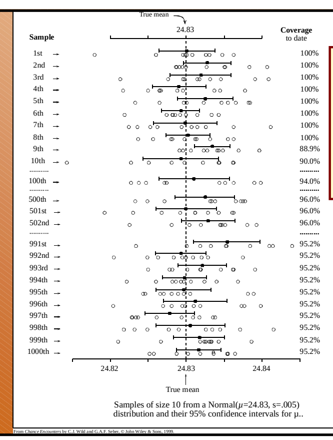
- CI are constructed using the sample $\bar{x}$ and s=SE. But diff. samples yield diff. estimates and ➔ diff. CI's?!?
- Below is a computer simulation showing how the process of taking samples effects the estimates and the CI's.



True mean almost always captured in the CI.

Samples of size 10 from a Normal(μ=24.83, s=.005) distribution and their 95% confidence intervals for µ..

Most of the table



How many of the previous samples contained the true mean?

Summary - CI for population mean

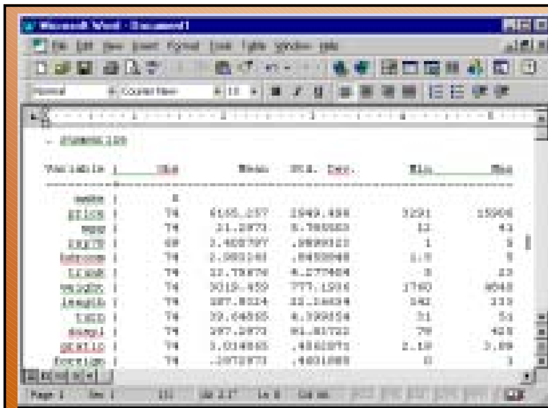
Confidence Interval for the true (population) mean μ:

sample mean $\pm$ ***t standard errors***

or $\quad \bar{x} \pm t\,\mathrm{se}(\bar{x}),\;$ where $\mathrm{se}(\bar{x}) = \dfrac{s_x}{\sqrt{n}}$ and $df = n-1$

Value of the Multiplier, *t*, for a 95% CI

df :	7	8	9	10	11	12	13	14	15	16	17
t :	2.365	2.306	2.262	2.228	2.201	2.179	2.160	2.145	2.131	2.120	2.110
df :	18	19	20	25	30	35	40	45	50	60	∞
t :	2.101	2.093	2.086	2.060	2.042	2.030	2.021	2.014	2.009	2.000	1.960



Output from other statistics packages

Minitab Output

T Confidence Intervals

```
Variable        N     Mean     StDev   SE Mean       95.0 % CI
Passage Times  20   24.8286   0.0051    0.0011  ( 24.8262, 24.8309)
```

Minitab *from menus*:
Stat → Basic Statistics → 1-Sample t
Check "Confidence interval" in dialogue box

Excel *from menus*:
Tools → Data Analysis
Choose "Descriptive Statistics",
Check "Summary" and "Confidence Level for Mean" in dialogue box

CI = 24.82855 ± 0.0023983

$\bar{x}$ → Mean; ± term for CI → Confidence Level(95.0%)

Selected Excel Output

Passage Times	
Mean	24.82855
Standard Error	0.0011459
. . . .	. . . .
Standard Deviation	0.0051245
. . . .	. . . .
Count	20
Confidence Level(95.0%)	0.0023983

Computer output for Newcomb's passage-time data.

Effect of increasing the confidence level

99% CI, $\bar{x} \pm 2.576\,\mathrm{se}(\bar{x})$

95% CI, $\bar{x} \pm 1.960\,\mathrm{se}(\bar{x})$

90% CI, $\bar{x} \pm 1.645\,\mathrm{se}(\bar{x})$

80% CI, $\bar{x} \pm 1.282\,\mathrm{se}(\bar{x})$

Confidence Level Increase

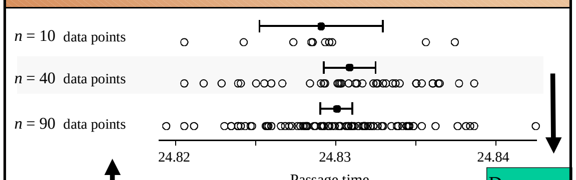
Increases the size of the CI

Why?

Figure 8.1.3 The greater the confidence level, the wider the interval

From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 2000.

Effect of increasing the sample size



Increase Sample Size

Decreases the size of the CI

Three random samples from a Normal(μ=24.83, s =.005) distribution and their 95% confidence intervals for μ.

From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 2000.

To ***double the precision*** we need ***four times*** as many observations.

Why ↑ in sample-size ↓ CI?

Confidence Interval for the true (population) mean μ:

sample mean $\pm$ ***t standard errors***

or $\quad \bar{x} \pm t\,\mathrm{se}(\bar{x}),\;$ where $\mathrm{se}(\bar{x}) = \dfrac{s_x}{\sqrt{n}}$ and $df = n-1$

CI for a population proportion

Confidence Interval for the true (population) proportion p:

sample proportion $\pm$ ***z standard errors***

or $\hat{p} \pm z\,\text{se}(\hat{p})$, where $\text{se}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, Section 7.3.

Example – higher blood thiol concentrations associated with rheumatoid arthritis?!?

TABLE 8.4.1 Thiol Concentration (mmol)

	Normal	Rheumatoid
	1.84	2.81
	1.92	4.06
	1.94	3.62
	1.92	3.27
	1.85	3.27
	1.91	3.76
	2.07	
Sample size	7	6
Sample mean	1.92143	3.46500
Sample standard deviation	0.07559	0.44049

Research question: Is the change in the Thiol status in the lysate of packed blood cells substantial to be indicative of a non trivial relationship between Thiol-levels and rheumatoid arthritis?

Example – higher blood thiol concentrations with rheumatoid arthritis

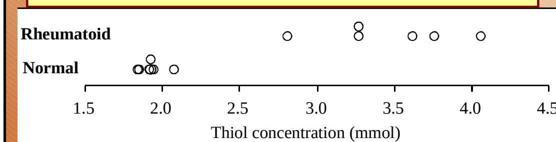


Figure 8.4.1 Dot plot of Thiol concentration data.

Two groups of subjects are studied: 1. NC (normal controls)
2. RA (rheumatoid arthritis).

Observations: 1. The avg. levels of thiol seem diff. in NC & RA
2. NC and RA groups are separated completely.

Question: Is there statistical evidence that thiol-level correlates with the disease?

Difference between means

Confidence Interval for a difference between population means ($\mu_1 - \mu_2$):

Difference between sample means

$\pm$ ***t standard errors of the difference***

or $\boxed{\bar{x}_1 - \bar{x}_2 \pm t\,\text{se}(\bar{x}_1 - \bar{x}_2)}$

Difference between proportions

Confidence Interval for a difference between population proportions ($p_1 - p_2$):

Difference between sample proportions

$\pm$ ***z standard errors of the difference***

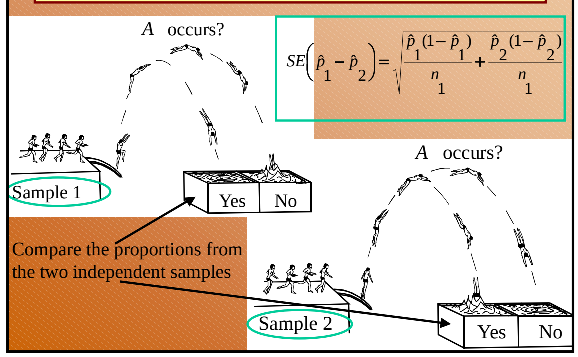
$\boxed{\hat{p}_1 - \hat{p}_2 \pm z\,\text{se}(\hat{p}_1 - \hat{p}_2)}$

Big Question ???

How do we compute the SE($\hat{p}_1 - \hat{p}_2$) for different cases?

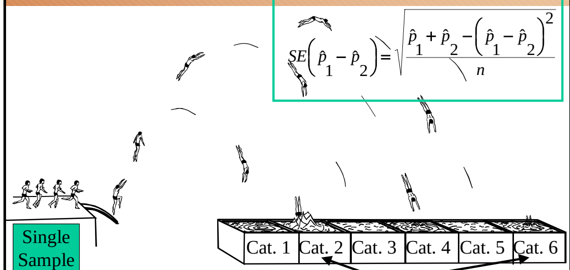
Proportions from 2 independent samples

$SE\left(\hat{p}_1 - \hat{p}_2\right) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_1}}$



Single sample, several response categories

$$SE\left(\hat{p}_1-\hat{p}_2\right)=\sqrt{\frac{\hat{p}_1+\hat{p}_2-\left(\hat{p}_1-\hat{p}_2\right)^2}{n}}$$



Compare different proportions from the same sample

Example – 1996 US Presidential Election

		Pre-election Polls				Election Results		
State	n	Clinton	Doll	Perot	Other/Undecided	Clinton	Doll	Perot
New Jersey	1,000	51	33	8	8	53	36	9
New York	1,000	59	25	7	9	59	31	8
Connecticutt	1,000	51	29	11	9	52	35	10

Compare proportions of NJ and NY voters supporting Clinton and Dole, pre- and post election

$$\hat{p}_1-\hat{p}_2 \pm z\,se(\hat{p}_1-\hat{p}_2)$$

Note the independence-case SE formula is only applicable for the cases when the samples are independent. In this case, the pre-election poll and the election results are **not independent** (obviously these are highly correlated observations).

Example – 1996 US Presidential Election

		Pre-election Polls				Election Results		
State	n	Clinton	Doll	Perot	Other/Undecided	Clinton	Doll	Perot
New Jersey	1,000	51	33	8	8	53	36	9
New York	1,000	59	25	7	9	59	31	8
Connecticutt	1,000	51	29	11	9	52	35	10

Single sample, several response categories

How far is Clinton ahead of Dole in NJ?
Diff.proportions= 18%
CI: [12% : 24%]
Actual diff 53-36=17

$$\hat{p}_1-\hat{p}_2 \pm z\,se(\hat{p}_1-\hat{p}_2)$$

$$\text{estimate} \pm z \times SE = \hat{p}_1-\hat{p}_2 \pm 1.96 \times SE\left(\hat{p}_1-\hat{p}_2\right)=$$

$$\hat{p}_1-\hat{p}_2 \pm 1.96\times\sqrt{\frac{\hat{p}_1+\hat{p}_2-\left(\hat{p}_1-\hat{p}_2\right)^2}{n}}=$$

$$0.18 \pm 1.96 \times 0.02842 = [12\% : 24\%]$$

Random sample of 1,000 people is taken from 5 countries to assess efficacy, cost and quality of health care

(Table entry is % agreeing)	Australia	Canada	N.Z.	UK	U.S.
Difficulties getting needed care	15	20	18	15	28
Recent changes will harm quality	28	46	38	12	18
System should be rebuilt	30	23	32	14	33
No bills not covered by insurance	7	27	12	44	8

2 independent Samples (n_1, n_2) compare proportions of people agreeing to a particular health care statement.

1 Sample, many response categories - compare proportions of New Zealanders either agreeing (Yes) or disagreeing (No) with a SET of statements.

SE's for the 3 cases of differences in proportion

(a) **Proportions from two independent samples** of sizes n_1 and n_2, respectively

$$se(\hat{p}_1-\hat{p}_2)=\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

(b) **One sample of size n, several response categories**

$$se(\hat{p}_1-\hat{p}_2)=\sqrt{\frac{\hat{p}_1+\hat{p}_2-(\hat{p}_1-\hat{p}_2)^2}{n}}$$

(c) **One sample of size n, many Yes/No items**

$$se(\hat{p}_1-\hat{p}_2)=\sqrt{\frac{Min(\hat{p}_1+\hat{p}_2,\hat{q}_1+\hat{q}_2)-(\hat{p}_1-\hat{p}_2)^2}{n}}$$

where $\hat{q}_1=1-\hat{p}_1$ and $\hat{q}_2=1-\hat{p}_2$

Sample size - proportion

- **For a 95% CI,** $\text{margin} = 1.96\times\sqrt{\hat{p}(1-\hat{p})/n}$
- **Sample size for a desired margin of error:**
  For a margin of error no greater than m, use a sample size of approximately
  $$n=\left(\frac{z}{m}\right)^2 \times p^*(1-p^*)$$
- p^* is a guess at the value of the proportion -- err on the side of being too close to 0.5
- z is the multiplier appropriate for the confidence level
- m is expressed as a proportion (between 0 and 1), not a percentage (basically, What's n, so that m >= margin?)

Sample size -- mean

- **Sample size for a desired margin of error:**
  For a margin of error no greater than m, use a sample size of approximately

$$n = \left(\frac{z\sigma^*}{m} \right)^2$$

- σ^* is an estimate of the variability of individual observations
- z is the multiplier appropriate for the confidence level

Chapter 8 Summary

Confidence intervals

- We construct an interval estimate of a parameter to summarize our level of uncertainty about its true value.
- The uncertainty is a consequence of the sampling variation in point estimates.
- If we use a method that produces intervals which contain the true value of a parameter for 95% of samples taken, the interval we have calculated from our data is called a 95% confidence interval for the parameter.
- Our confidence in the particular interval comes from the fact that the method works 95% of the time (for 95% CI's).

TABLE 8.7.1 Standard Errors and Degrees of Freedom

Parameter		Estimate	Standard error of estimate	*df*
Mean,	μ	$\bar{x}$	$\frac{s_x}{\sqrt{n}}$	*n-1*
Proportion,	p	$\hat{p}$	$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	∞
Difference in means,	$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	Min(n_1-1,n_2-1)
Difference in proportions,	p_1-p_2	$\hat{p}_1 - \hat{p}_2$	(see Table 8.5.5)	∞

$df = \infty$ means we use a muitiplier obtained from the Normal(0,1) distribution.
CI's work well when sample sizes are big enough to satisfy the 10% rule in Appendix A3.
Applies to means from independent samples.
df given is a conservative approximation for hand calculation (see Section 10.2).

Summary cont.

- For a great many situations,

  an (approximate) confidence interval is given by

  estimate $\pm$ ***t standard errors***

  The size of the multiplier, t, depends both on the desired confidence level and the degrees of freedom (df).

  [With proportions, we use the Normal distribution (i.e., $df = \infty$) and it is conventional to use z rather than t to denote the multiplier.]
- The *margin of error* is the quantity added to and subtracted from the estimate to construct the interval (i.e. t standard errors).

Summary cont.

- If we want greater confidence that an interval calculated from our data will contain the true value, we have to use a wider interval.
- To double the precision of a 95% confidence interval (i.e.halve the width of the confidence interval), we need to take 4 times as many observations.