STAT 110 A, Probability & Statistics for Engineers I

UCLA Statistics, Spring 2003

http://www.stat.ucla.edu/~dinov/courses_students.html

SOLOTION HOMEWORK 2

Due Date: Friday, Apr. 25, 2003

http://www.stat.ucla.edu/%7Edinov/courses_students.dir/03/Spr/Stat110A.dir/HWs.dir/HW1.html

Assignment 2 Solution (There is a total of 100 points for this assignment.)

Problem 1

a) (5)(4)=20 (4 points)

b) (5)(4)(3)=60 (4 points)

c) $\binom{5}{2} = \frac{5!}{2!3!} = 10$ (4 points)

Problem2

a) $\binom{20}{5} = \frac{20!}{5!15!} = 15504$ (3 points)

b) $\binom{8}{4}\binom{12}{1} = 840$ (3 points)

c) $\binom{8}{4}\binom{12}{1} \Big/ \binom{20}{5} = 840/15504 = 0.0542$ (3 points)

d) P(at least 4) = P(exactly 4)+P(exactly 5) = $\binom{8}{4}\binom{12}{1} \Big/ \binom{20}{5} + \binom{8}{5}\binom{12}{0} \Big/ \binom{20}{5}$

= 0.0542 + 0.0036 = 0.0578 (3 points)

Problem 3

a) If the A's are distinguishable from one another and similarly for the B's, C's, and D's,

then there are 12! possible chain molecules; otherwise, there are $\frac{12!}{3!3!3!3!} = 369600$

possible chain molecules. (8 points)

b) Think of the group of 3 A's as a single molecule and similarly for the B's, C's, and D's. Then there are 4! ways to order these entities. Thus,

P(all together) = $\frac{4!}{369600} = 0.00006494$ (8 points)

Problem 4

a) P(A) = 0.15 + 0.10 + 0.10 + 0.10 = 0.45 (2 points)

P(B) = 0.10 + 0.15 = 0.25 (2 points)

$P(A \cap B) = 0.10$ (2 points)

b) $P(A|B) = P(A \cap B)/P(B) = (0.10)/(0.25) = 0.40$ (2 points)

Knowing that the car is black, the probability that it has automatic transmission is 0.40. (1 point)

$P(B|A) = P(A \cap B)/P(A) = (0.10)/(0.45) = 0.2222$ (2 points)

Knowing that the car has automatic transmission, the probability that it is black is 0.2222. (1 point)

c) $P(A|C) = P(A \cap C)/P(C) = (0.15)/(0.30) = 0.50$ (2 points)

The probability that car has automatic transmission given that the car is white is 0.50. (1 point)

$P(A|C') = P(A \cap C')/P(C') = (0.15)/(0.70) = 0.2143$ (2 points)

Knowing that the car is not white, the probability that it has automatic transmission is 0.2143. (1 point)

Problem 5

a) P(both are O) = (0.44)(0.44) = 0.1936 (5 points)

b) $P(\text{two individuals match}) = 0.42^2 + 0.10^2 + 0.04^2 + 0.44^2 = 0.3816$ (5 points)

Problem 6

Let A_1 = older pump fails, A_2 = newer pump fails, and $X = P(A_1 \cap A_2)$. Then

$P(A_1) = 0.10 + X$ (3 points for equation)

$P(A_2) = 0.05 + X$ (3 points for equation)

$X = P(A_1 \cap A_2) = P(A_1) \times P(A_2) = (0.10 + X)(0.05 + X)$ (4 points for equation)

The resulting quadratic equation has roots X = 0.0059 (3 points) and X = 0.8441 (3 points). Hopefully the smaller root is the actual probability of system failure.

Problem 7

Let q denote the probability that a rivet is defective.

a) P (seam need rework) = 0.14 = 1 – P(seam doesn't need rework)

= 1 – P(no rivets are dective)

= $1 - (1 - q)^{25}$ (4 points for equation)

q = 0.00601 (4 points)

b) $0.10 = 1 - (1 - q)^{25}$ (4 points for equation)

q = 0.00421 (4 points)