STAT 110 A, Probability & Statistics for Engineers I

UCLA Statistics, Spring 2003

http://www.stat.ucla.edu/~dinov/courses_students.html

SOLOTION HOMEWORK 3

Due Date: Friday, May 09, 2003

http://www.stat.ucla.edu/%7Edinov/courses_students.dir/03/Spr/Stat110A.dir/HWs.dir/HW3.html

Assignment 3 Solution (There is a total of 100 points for this assignment.)

Problem 1

There is no 00000, nor any zip codes with four zeros. Thus possible X values are 2, 3, 4, and 5. (4 points) X = 0 for 15213 (4 points), X = 4 for 44074 (4 points), and X = 3 for 90024. (4 points) (Any other examples are also okay.)

Problem 2

a) $\sum p(x) \neq 1$ for 1st and 3rd p(x). (4 points)

b) $P(2 \leq X \leq 4) = p(2) + p(3) + p(4) = 0.5$ (4 points)

$P(X \leq 2) = p(0) + p(1) + p(2) = 0.6$ (4 points)

$P(X \neq 0) = 1 - p(0) = 0.6$ (4 points)

c) $\sum_{x=0}^{4} p(x) = c[(5-0) + (5-1) + (5-2) + (5-3) + (5-4)] = 1 \Rightarrow c = \frac{1}{15}$ (4 points)

Problem 3

a) $P X \leq 3) = p(0) + p(1) + p(2) + p(3) = 0.10 + 0.15 + 0.20 + 0.25 = 0.70$ (4 points)

b) $P(X < 3) = P(x \leq 2) = p(0) + p(1) + p(2) = 0.45$ (4 points)

c) $P(3 \leq X) = p(3) + p(4) + p(5) + p(6) = 0.55$ (4 points)

d) $P(2 \leq X \leq 5) = p(2) + p(3) + p(4) + p(5) = 0.71$ (4 points)

e) The number of lines not is use is 6 – X, so 6 – X = 2 is equivalent to X = 4, 6 – X = 3 to x =3, and 6 – x = 4 to x = 2. Thus $P(2 \leq X \leq 4) = p(2) + p(3) + p(4) = 0.65$ (4 points)

f) $6 - X \geq 4$ if $6 - 4 \geq X$, ie, $X \leq 2$, and $P(X \leq 2) = 0.45$ (4 points)

Problem 4

a) $E(X) = \sum_{x=0}^{4} xp(x) = (0)(0.08) + (1)(0.15) + (2)(0.45) + (3)(0.27) + (4)(0.05)$

$= 2.06$ (4 points)

b) $V(X) = \sum_{x=0}^{4}(x - 2.06)^2 p(x) = (0 - 2.06)^2(0.08) + \ldots + (4 - 2.06)^2(0.05)$

$= 0.9364$ (4 points)

c) $\sigma_x = \sqrt{0.9364} = 0.9677$ (4 points)

d) $V(X) = \left[\sum_{x=0}^{4} x^2 p(x)\right] - (2.06)^2 = 5.18 - 4.2436 = 0.9364$ (4 points)

Problem 5

a) $E(X) = (13.5)(0.2) + (15.9)(0.5) + (19.1)(0.3) = 16.38$ (4 points)
$E(X^2) = (13.5)^2(0.2) + (15.9)^2(0.5) + (19.1)^2(0.3) = 272.298$ (4 points)
$V(X) = 272.298 - (16.38)^2 = 3.9936$ (4 points)

b) $E(25X - 8.5) = 25E(X) - 8.5 = (25)(16.38) - 8.5 = 401$ (4 points)

c) $V(25X - 8.5) = V(25X) = (25)^2V(X) = (625)(3.9936) = 2496$ (4 points)

d) $E[h(X)] = E[X - 0.01X^2] = E(X) - 0.01E(X^2) = 16.38 - 2.72 = 13.66$ (4 points)