STAT 110 A, Probability & Statistics for Engineers I

UCLA Statistics, Spring 2003

http://www.stat.ucla.edu/~dinov/courses_students.html

SOLOTION HOMEWORK 4

Due Date: Friday, May 23, 2003

http://www.stat.ucla.edu/%7Edinov/courses_students.dir/03/Spr/Stat110A.dir/HWs.dir/HW4.html

Assignment 4 Solution (There is a total of 100 points for this assignment.)

Problem1 (Total: 16 points; 4 points each)

a) Let x = number of male children before the second female child. Then
P(X = x) = nb(x;2,0.5)
b) P(exactly 4 children) = P(exactly 2 males) = nb(2;2,0.5) = 0.188
c) P(at most 4 children) = $P(X \le 2) = \sum_{x=0}^{2} nb(x,2,0.5) = 0.688$
d) E(X) = (2)(0.5)/(0.5) = 2,
expected number of children = E(X + 2) = E(X) + 2 = 4

Problem 2 (Total: 9 points; 3 points each)

a) $P(X = 1) = e^{-0.2}(0.2) = 0.1637$
b) $P(X \ge 2) = 1 - P(X = 0) - P(X = 1) = 1 - 0.9825 = 0.0175$
c) $P(X = 0)\cdot P(X = 0) = (0.8187)^2 = 0.6703$

Problem 3 (Total: 20 points; 4 points each)

a) $f(x) = \frac{1}{10}$ $-5 \le x \le 5$ $\qquad F(x) = \begin{cases} 0 & x < -5 \\ \frac{x+5}{10} & -5 \le x < 5 \\ 1 & x \ge 5 \end{cases}$

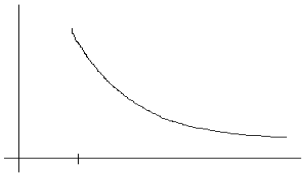
b) $P(X < 0) = \int_{-5}^{0} \frac{1}{10} dx = 0.5$ $\qquad$ c) $P(-2.5 < X < 2.5) = \int_{-2.5}^{2.5} \frac{1}{10} dx = 0.5$

d) $P(-2 \le X \le 3) = 0.5$

e) $P(k < X < k+4) = \int_k^{k+4} \frac{1}{10} dx = \frac{1}{10}\left[(k+4) - k\right] = 0.4$

Problem 4 (Total: 24 points; 3 points each)

a)



b) $\int_\theta^\infty \frac{k\theta^k}{x^{k+1}} dx = \theta^k \left(-\frac{1}{x^k} \right)_\theta^\infty = \frac{\theta^k}{\theta^k} = 1$

c) $P(X \le b) = \int_\theta^b \frac{k\theta^k}{x^{k+1}} dx = 1 - \left(\frac{\theta}{b}\right)^k$

d) $P(a \le X \le b) = \int_a^b \frac{k\theta^k}{x^{k+1}} dx = \left(\frac{\theta}{a}\right)^k - \left(\frac{\theta}{b}\right)^k$

e) $E(X) = \int_\theta^\infty x \frac{k\theta^k}{x^{k+1}} dx = \frac{k\theta}{k-1}$

f) $E(X) = \infty$

g) $E(X^2) = k\theta^k \int_\theta^\infty \frac{1}{x^{k-1}} dx = \frac{k\theta^2}{k-2}$ so $\mathrm{Var}(X) = \frac{k\theta^2}{k-2} - \left(\frac{k\theta}{k-1}\right)^2 = \frac{k\theta^2}{(k-2)(k-1)^2}$

h) $\mathrm{Var}(X) = \infty$ since $E(X^2) = \infty$

Problem 5 (Total: 15 points; 3 points each)

a.1) $P(X \ge 10) = P(Z \ge 0.43) = 1 - \Phi(0.43) = 1 - 0.6664 = 0.3336$

a.2) $P(X > 10) = P(X \ge 10) = 0.3336$

b) $P(X > 20) = P(Z > 4) \approx 0$

c) $P(5 \le X \le 10) = P(-1.36 \le Z \le 0.43) = \Phi(0.43) - \Phi(-1.36) = 0.6664 - 0.0869 = 0.5795$

d) $P(8.8 - c < X < 8.8 + c) = 0.98$

$8.8 - c = \mu + (-2.33)\sigma = 8.8 - 2.33(2.8) \Rightarrow c = 2.33(2.8) = 6.524$

Problem 6 (Total: 16 points; 4 points each)

a) $P(67 \le X \le 75) = P(-1 \le Z \le 1.67) = 0.7938$

b) $P(70 - c \le X \le 70 + c) = 0.95$

$70 - c = \mu + (-1.96)\sigma = 70 - 1.96(3) \Rightarrow c = 1.96(3) = 5.88$

c) $10 \cdot P(\text{a single one is acceptable}) = 10(0.7938) = 7.938$

d) $P(X < 73.84) = P(Z < 1.28) = 0.9$ so $P(Y \leq 8) = B(8;10,0.9) = 0.264$