# STAT 110 A, Probability & Statistics for Engineers I

## UCLA Statistics, Spring 2003

http://www.stat.ucla.edu/~dinov/courses_students.html

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# SOLUTION HOMEWORK 5

## Due Date: Friday, June 06, 2003

http://www.stat.ucla.edu/%7Edinov/courses_students.dir/03/Spr/Stat110A.dir/HWs.dir/HW5.html

(There is a total of 100 points for this assignment.)

**Problem1 (Total: 16 points; 4 points each)**

a) Yes.

b) $\{X \geq t\} = A1 \cap A2 \cap A3 \cap A4 \cap A5$

c) $P(X \geq t) = P(A1)P(A2)P(A3)P(A4)P(A5) = \left(e^{-\lambda t}\right)^5 = e^{-0.05t}$

$F(t) = P(X \leq t) = 1 - e^{-0.05t}$

d) $f(t) = 0.05\, e^{-0.05t}$, X has an exponential distribution with $\lambda = 0.05$.

**Problem 2 (Total: 16 points; 4 points each)**

X~ normal distribution; P = 0.1; n=200; np = 20; npq =18

a) $P(X \leq 30) = \Phi\left(\frac{30 + 0.5 - 20}{\sqrt{18}}\right) = 0.9932$

b) $P(X < 30) = P(X \leq 29) = \Phi(2.24) = 0.9875$

c) $P(15 \leq X \leq 25) = P(X \leq 25) - P(X \leq 14) = \Phi(1.30) - \Phi(-1.30) = 0.8064$

d) $X = 20 - 0.44\,(\sqrt{18}) = 18.1332$

**Problem 3 (Total: 15 points; 5 points each)**

a) $f(y|x) = \frac{f(x,y)}{f_x(x)} = \frac{\frac{9}{26} y^2 (x+1)^2}{\int_0^1 \frac{9}{26} y^2 (x+1)^2 dy} = 3y^2$

b) $P(Y < ½ \mid X < ½) = \dfrac{P(0 < x < 1/2; 0 < y < 1/2)}{P(0 < x < 1/2)} = \dfrac{\int_0^{1/2}\int_0^{1/2} f(x,y)dydx}{\int_0^{1/2} f_x(x)dx} = \dfrac{1}{8}$

c) $E(Y \mid X = x) = \int y f(y \mid x)dy = \int_0^1 y(3y^2)dy = \dfrac{3}{4}$

## Problem4 (Total: 16 points; 4 points each)

Refer to your lecture notes.

## Problem5 (Total: 14 points; 7 points each)

a) $E(X) = \int_0^1 x(\theta+1)x^\theta dx = \dfrac{\theta+1}{\theta+2} = 1 - \dfrac{1}{\theta+2}$ So $\overline{X} = 1 - \dfrac{1}{\theta+2} \Rightarrow \hat{\theta} = \dfrac{1}{1-\overline{X}} - 2$

Since $\overline{X} = 0.80$, $\hat{\theta} = 3$.

b) $f(x_1,\ldots,x_n;\theta) = (\theta+1)^n (x_1 x_2 \cdots x_n)^\theta$ so $\log f = n\ln(\theta + 1) + \theta\sum\ln(x_i)$

$\Rightarrow \hat{\theta} = -\dfrac{n}{\sum \ln(x_i)} - 1$ so $\hat{\theta} = 3.12$

## Problem 6 (Total: 10 points)

95% C.I. is $1.95 \pm 2.365(0.22/\sqrt{8}) = (1.766, 2.134)$

## Problem 7 (Total: 13 points)

p = 260/500 = 0.52

s.d. = $\sqrt{\dfrac{pq}{n}} = \sqrt{\dfrac{(0.52)(0.48)}{500}} = 0.02234$ (1 point)

95% C.I. is $0.52 \pm (1.96)(0.02234) = (0.476, 0.564)$ ( 6 points)

$0.02 = \sqrt{\dfrac{(0.52)(0.48)}{n}}$ so n = 624 (6 points)