Stats 110B

HW2 Suggested Solutions

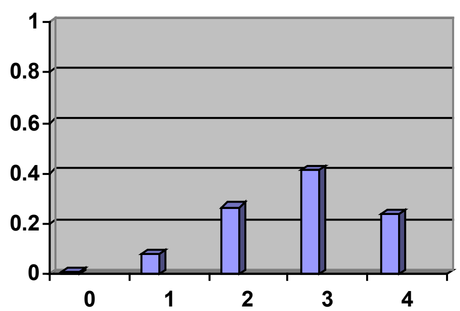
http://www.stat.ucla.edu/~dinov/courses_students.dir/03/Spr/Stat110B.dir/STAT110B.html

http://www.stat.ucla.edu/~dinov/courses_students.dir/03/Spr/Stat110B.dir/assignments.html

1. (a)

Outcome	Probability	X
SSSS	0.3^4 = 0.0081	4
FSSS	0.3^3*0.7 = 0.0189	3
SFSS	0.3^3*0.7 = 0.0189	3
SSFS	0.3^3*0.7 = 0.0189	3
SSSF	0.3^3*0.7 = 0.0189	3
FFSS	0.3^2*0.7^2 = 0.0441	2
SSFF	0.3^2*0.7^2 = 0.0441	2
SFSF	0.3^2*0.7^2 = 0.0441	2
FSFS	0.3^2*0.7^2 = 0.0441	2
SFFS	0.3^2*0.7^2 = 0.0441	2
FSSF	0.3^2*0.7^2 = 0.0441	2
SFFF	0.3*0.7^3 = 0.1029	1
FSFF	0.3*0.7^3 = 0.1029	1
FFSF	0.3*0.7^3 = 0.1029	1
FFFS	0.3*0.7^3 = 0.1029	1
FFFF	0.7^4 = 0.2401	0

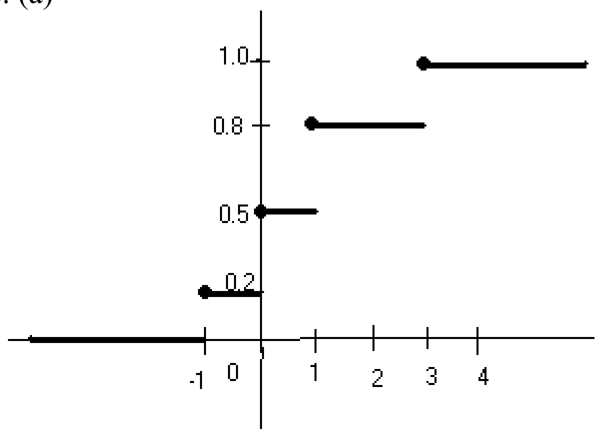
(b)



(c) expected value = mean = 4*0.3 = 1.2. Most likely value is the value with the largest probability, i.e., 3.

(d) P(at least two have insurance) = $P(X\geq 2) = 1 - P(X=0) - P(X=1)$
$= 1 - 0.2401 - 0.4116 = 0.3483$

2. (a)



(b) pdf :

y	-1	0	1	3
f(y)	0.2	0.3	0.3	0.2

(c) $P(0 \leq Y \leq 2) = P(Y=0)+P(Y=1) = 0.6$

(d) mean = $(-1\times0.2+0\times0.3+1\times0.3+3\times0.2) = 0.7$

Variance = $(-1 - 0.7)^2\times0.2+(0 - 0.7)^2\times0.3+(1 - 0.7)^2\times0.3+(3 - 0.7)^2\times0.2$

$= 1.81$

(e) $Z = e^{\sin(\pi Y/2)}$

y	-1	0	1	3
Z	0.36788	1	2.71828	0.36788

Therefore pdf for Z

Z	0.3688	1	2.71828
P(Z)	0.4	0.3	0.3

$\mu_z = 0.3688\times0.4 + 1\times0.3 + 2.71828\times0.3 = 1.263$

Variance = $(0.3688 - 1.263)^2\times0.4 + (1 - 1.263)^2\times0.3 + (2.71828 - 1.263)^2\times0.3 = 0.9759$

$\mu_z/\sigma_z = 1.263/(0.9759)$^0.5 = 1.2785

3. Assume that the number of calls per hour follows a Poisson distribution.

$\lambda = 2$ per hour = 0.5 per 15mins

(a) P(no calls) = $P(x = 0)$

$= (0.5^0 \times e^{-0.5})/0! = 0.6065$

(b) P(no more than I call) = $P(x\leq1) = P(x=0) + P(x=1)$

$= (0.5^0 \times e^{-0.5})/0! + (0.5^1 \times e^{-0.5})/1!$

$= 0.9098$

4. (a) X has a normal distribution.

pdf : $f(x) = 1/(5\sqrt{2\pi})e^{-(X-15)^2/(2\times25)} = 1/(5\sqrt{2\pi})e^{-(X-15)^2/50}$

(b) P(reach printer within 3 mins) = $P(0 \le X \le 3)$

$= P(X \le 3) - P(X \le 0) = P(Z \le (3-15)/5) - P(Z \le -3) = P(Z \le -2.4) = 0.0082 - 0.0013$

$= 0.0069$

(c) $P(10 \le X \le 20) = P((10-15)/5 \le Z \le (20-15)/5) = P(-1 \le Z \le 1)$

Since the probability of getting a value within one standard deviation of the mean is approximately 68%, it is not unusual for us to stay in queue between 10 and 20mins.

(d) $P(X > 30) = P(Z > (30-15)5) = P(Z > 3) = 1 - P(X \le 3) \approx 0$

Since the probability is close to zero, it would be unusual to take more than 30 mins.

5. (a) Since the sample size is large, we can use normal approximation.

mean = 1600(0.5) = 800, variance = 1600(0.5)(1 – 0.5) = 400

Let number of students coming to UCLA be X

P(UCLA ends up with more students than it can accommodate)

$= P(X > 820) = P(Z > (820 - 800)/20) = P(Z > 1) = 1 - P(X \le 1)$

$= 1 - 0.8413 = 0.1587$

(b) Given P(getting more than UCLA could accommodate) = 0.05

$= P(Z \ge 1.65) = P((820 - \mu_n)/20 \ge 1.65)$

where μ_n is the expected number of student coming to UCLA

solve for μ_n, $\mu_n = 787$; $\mu_n = 0.5(n)$, so n must be $787\times2 = 1574$

6. (a) $Var(X+Y) = Var(x) + Var(Y) + 2Cov(X,Y)$

$= E(X^2) - E(X)^2 + E(Y^2) - E(Y)^2 + E(XY) - E(X)E(Y)$

$= (41 - 16) + (10 - 1) + 2(6 - (-4)) = 54$

(b) correlation coefficient $\rho = Cov(X,Y)/(\sigma_X\sigma_Y) = 10/(5*3) = 2/3$

(c) If they are independent, Cov(X,Y) must be 0. Now Cov =10, therefore they are not independent.