STAT 110 B, Probability & Statistics for Engineers II
UCLA Statistics, Spring 2003

http://www.stat.ucla.edu/~dinov/courses_students.html

HOMEWORK 4

Due Date: Friday, May 23, 2003, turn in after lecture

Correct solutions to any five problems carry full credit. See the HW submission rules. On the front page include the following header.

- (**HW_4_1**) [sec. 7.1, #10] A random sample of n=15 heat pumps of a certain type yielded the following observations on lifetime in years:

2.0	1.3	6.0	1.9	5.1	0.4	1.0	5.3	15.7	0.7	4.8	0.9	12.2	5.3	0.6

(a) Assume the lifetime distribution is Exponential and use an argument parallel to that of Example 7.5 (p. 284) to obtain a 95% confidence interval for the expected (true average) lifetime.
(b) How should the interval in part (a) be altered to achieve a confidence level of 99%? Would this CI be larger than or smaller than the first one?
(c) What is a 95% CI(σ), where σ is the standard deviation of the lifetime distribution [Hint: What is the standard deviation for Exponential distribution?]

- (**HW_4_2**) [sec. 7.2, #25] A state legislator wishes to survey residents of her district to see what proportion of the electorate is aware of her position on using state funds to pay for abortions.

69.5	71.9	72.6	73.1	73.3	73.5	75.5	75.7	75.8	76.1	76.2	76.2	77.0	77.9	78.1	79.6	79.7	79.9	80.1	82.2	83.7	93.7

Calculate a 99% CI(), where is the standard deviation of the fracture toughness distribution. Is this interval valid whatever the nature of the distribution? Explain!

- (**HW_4_3**) [sec. 8.1, #9] Two different companies have applied to provide cable television service in a certain region. Let p denote
the proportion of all potential subscribers who favor the first company over the second. Consider testing H_o: p = 0.5 versus H_a: p ≠ 0.5 based on a random sample of 25 individuals. Let X denote the number in the sample who favor the first company and x represent the observed value of X.
(a) Which of the following rejection regions is most appropriate and why?
$R_1 = \{x : x \leq 7 \text{ or } x \geq 18\}$; $R_2 = \{x : x \leq 8\}$; $R_3 = \{x : x \geq 17\}$
(b) In the context of this problem situation, describe what type I and type II errors are.
(c) What is the probability distribution of the test statistic X when H_o is true? Use it to compute the probability of a type I error.
(d) Compute the probability of a type II error for the selected region for each of the alternatives p = 0.3, p = 0.4, p = 0.6 and p = 0.7.
(e) Using the selected region, what would you conclude if 6 of the 25 queried favored company 1?

- (**HW_4_4**) [sec. 8.1, #11] The calibration of a scale is to be checked by weighing a 10-kg test specimen 25 times.

Suppose that the results of different weightings are independent of one another and that the weight on each trial is Normally distributed with $\sigma = 0.200$kg. Let μ denote the true average weight reading on the scale.
(a) What hypotheses should be tested?
(b) Suppose the scale is to be recalibrated if the sample average, $\bar{x}$, is either $\bar{x} \geq 10.1032$ or $\bar{x} \leq 9.8968$. What is the probability that recalibration is carried out when it is actually unnecessary?
(c) What is the probability that recalibration is judged unnecessary when in fact μ=10.1? When μ= 9.8?
(d) For a significance level of $\alpha = 0.05$ what would be the appropriate rejection region?

- (**HW_4_5**) [sec. 9.2, #23] Fusible interlinings are being used with increasing frequency to support outer fabrics and improve the shape of various pieces of clothing. The article Computability *of Outer and Fusible Interlining Fabrics in Tailored Garments (Textile Res. J.*, 1997: 137-142) gave the accompanying data on extensibility (%) at 100 gm/cm for both high-quality (H) and poor-quality (P) fabric specimens:

H	1.2	1.9	0.7	1.0	1.7	1.7	1.1	0.9	1.7	1.9	1.3	2.1	1.6	1.8	1.4	1.3	1.9	1.6	0.8	2.0	1.7	1.6	2.3	2.0
P	1.6	1.5	1.1	2.1	1.5	1.3	1.0	2.6																

(a) Construct normal probability plots to verify the plausibility of both samples having been selected from Normal population distributions.
(b) Construct a comparative box-plot. Does it suggest that there is a difference between the true average extensibility for high-quality fabric and that for poor-quality fabric?
(c) The sample mean and standard deviation of the H sample are: 1.508 and 0.444, and those of the P sample are: 1.588 and 0.530, respectively. Use a two-sample T-test to determine whether true average extensibility differs for the two types of fabrics.

- (**HW_4_6**) [sec. 6.2, #22] Let X denote the proportion of alotted time that s randomly selected student spends working on a certain aptitude test. Suppose the pdf of X is

$$f(x; \vartheta) = \begin{cases} (\vartheta + 1)\, x^{\vartheta} & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\vartheta > -1$. A random sample of 10 students yields the following data

0.92	0.79	0.90	0.65	0.86	0.47	0.73	0.97	0.94	0.77

(a) Use the ***method of moments*** to obtain an estimator of ϑ and then use this to compute an actual estimate for these data.
(b) Obtain a ***maximum-likelihood estimator*** of ϑ and use it to calculate an estimate for the given data.

Last modified on by dinov@stat.ucla.edu.

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