Stats 110B

HW4 Suggested Solutions

http://www.stat.ucla.edu/~dinov/courses_students.dir/03/Spr/Stat110B.dir/STAT110B.html
http://www.stat.ucla.edu/~dinov/courses_students.dir/03/Spr/Stat110B.dir/assignments.html

1. (a) df = 2n = 30
   $2\lambda\Sigma X_i$ has a chi-squared distribution with df =30,
   $\chi^2_{0.025} = 46.979$, $\chi^2_{0.975} = 16.791$
   $P(16.791 < 2\lambda\Sigma X_i < 46.979) = 0.95$
   $P((2\lambda\Sigma X_i)/46.979 < 1/\lambda < (2\lambda\Sigma X_i)/16.791) = 0.95$
   $\Sigma X_i = 63.2$
   the 95% CI = (2.96, 7.53)
   (b) use $\chi^2_{0.995} = 13.787$ and $\chi^2_{0.005} = 53.672$ instead. The interval would be wider.
   (c) Since the distribution is exponential, $\sigma = \mu = 1/\lambda$. Therefore the 95% CI for σ is the same as that of the mean calculated in part (a).

2. variance = 25.368, n = 22
   99% CI for variance = $((21)25.368/\chi^2_{0.005,\,21}\ ,\ (21)25.368/\chi^2_{0.995,\,21})$
   = (12.87, 66.317)
   99% CI for SD = $(12.87^{1/2}, 66.317^{1/2})$
   = (3.59, 8.14)
   The distribution has to be normal and independent.

3. (a) $R_1 = \{x : x \le 7 \text{ or } x \ge 18\}$ since the test is a two-sided test
   (b) Type I error : judging one of the two companies favored over the other when in fact the split of the market is half-half.
   Type II error : judging the split is half-half when it is not.
   (c) X has a binomial distribution with n = 25 and p = 0.5
   P(Type I error) = $P(x \le 7 \text{ or } x \ge 18 \text{ when } p = 0.5)$
   = 0.044
   (d) $\beta(0.4) = P(8 \le x \le 17 \text{ when } p = 0.4) = 0.845$. $\beta(0.6) = 0.845$
   similarly $\beta(0.3) = \beta(0.7) = 0.488$
   (e) Since x = 6 is in the rejection region, we would reject H_o and conclude that the split is not even.

4. (a) $H_o : \mu = 10$
   $H_a : \mu \neq 10$
   (b) P(recalibration is carried out when it is actually unnecessary)
   = P(Type I error)
   = $P(\bar{x} \ge 10.1032 \text{ or } \bar{x} \le 9.8968 \text{ when } \mu = 10)$
   = $P(z \ge (10.1032 - 10)/(0.2/5) \text{ or } z \le (9.8968 - 10)/(0.2/5))$

$= P(z \geq 2.58 \text{ or } z \leq -2.58) = 0.0049 + 0.0049 = 0.0098$

(c) P(recalibration is judged unnecessary when in fact $\mu = 10.1$)

$= P(\text{Type II error})$

$= P(9.8968 < \bar{x} < 10.1032 \text{ when } \mu = 10.1)$

$= P(-5.08 < z < 0.08) = 0.5319$

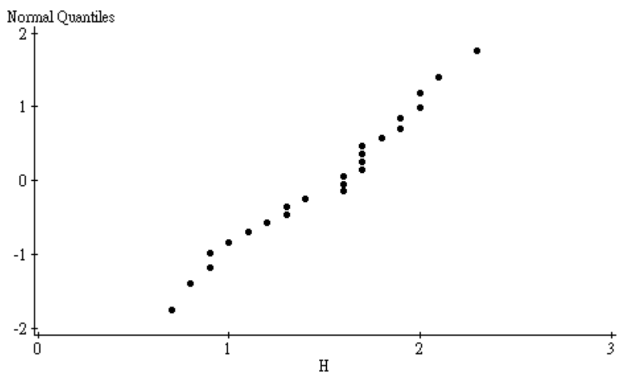
Similarly for $\mu = 9.8$

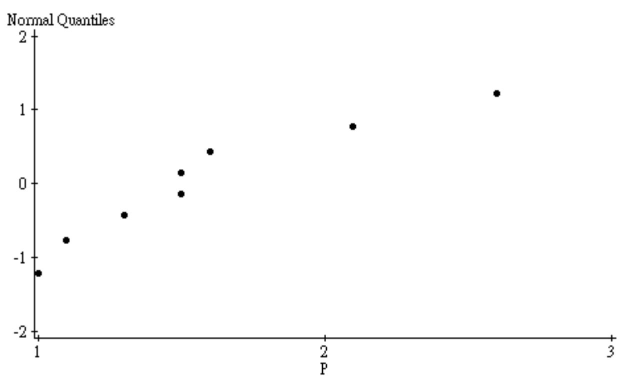
$P(2.42 < z < 7.58) = 0.0078$

(d) $\alpha = 0.05$, rejection region should be $z \leq -1.96$ or $z \geq 1.96$

i.e. $\bar{x} \leq 9.9216$ or $\bar{x} \geq 10.0784$

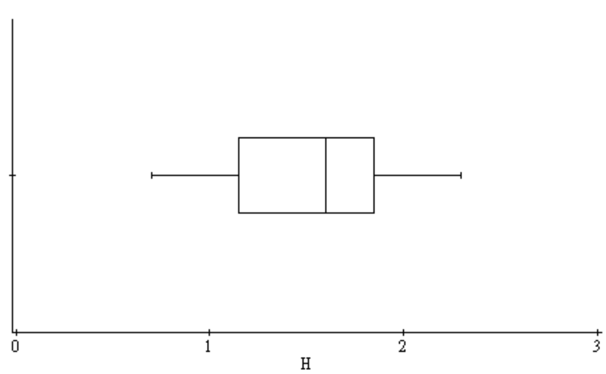
5. (a) Normal probability plots

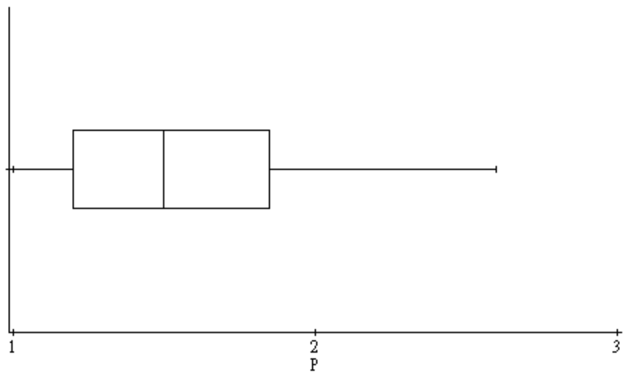




The points in both plots generally form a straight line. Therefore the assumption of normal distribution is not violated

(b) Boxplots :





The boxplots do not suggest a difference

(c) $H_o : \mu_1 - \mu_2 = 0$

$H_a : \mu_1 - \mu_2 \neq 0$

μ_1 = population mean of H

μ_2 = population mean of P

The two samples are independent, we use df = min(24-1, 8-1) = 7, $\alpha = 0.05$

$t = (1.508 - 1.588)/(0.444^2/24 + 0.53^2/8)^{1/2}$

$= -0.3843 < t_{0.025,\,7}$

Therefore we do not reject the null hypothesis and conclude that there is not enough evidence to claim that the true average extensibility differs for the two types of fabrics.

6. $f(x, \vartheta) = (\vartheta + 1)x^\vartheta$ for $0 \leq x \leq 1$

(a) method of moments :

$E(x) = \int_0^1 x(\vartheta + 1)x^\vartheta dx = (\vartheta + 1)/ (\vartheta + 2)$

$= 1 - 1/(\vartheta + 2)$

estimate of $\vartheta = 1/(1 - \bar{x}) - 2$

$\bar{x} = 0.8$

estimate of $\vartheta = 3$

(b) Maximum-likelihood :

$f(x_1, \ldots ,x_n, \vartheta) = (\vartheta + 1)^n(x_1, \ldots ,x_n)^\vartheta$

log likelihood

$l(x) = n \ln(\vartheta + 1) + \vartheta\Sigma\ln(x_i)$

$d\, l/d\vartheta = 0$

$n/(\vartheta + 1) = -\Sigma\ln(x_i)$

estimate of $\vartheta = -n/\Sigma\ln(x_i) - 1$

$\Sigma\ln(x_i) = -2.4295$, n =10

estimate of $\vartheta = 3.116$