AIU FOR 6520

Statistical Research Design & Methods in Forensic Psychology

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http://www.stat.ucla.edu/~dinov/courses_students.html

Sensitivity vs. Specificity

● Sensitivity is a measure of the fraction of gold standard known examples that are correctly classified/identified.

● **Sensitivity= TP/(TP+FN)**

● Specificity is a measure of the fraction of negative examples that are correctly classified:

● **Specificity = TN/(TN+FP)**

●TP = True Positives

●FN = False Negatives

●TN = True Negatives

●FP = False Positives

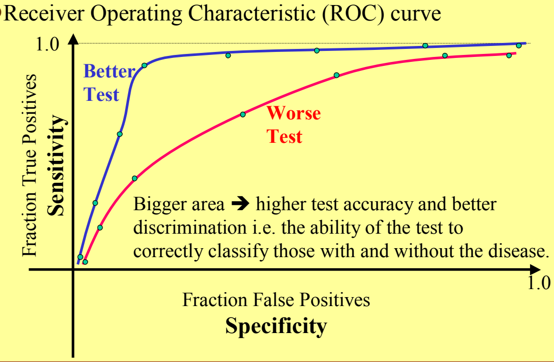
H_o: no effects (μ=0)

True Reality

Test Results	H_o true	H_o false
Can't reject	TN	FN
Reject H_o	FP	TP

The ROC Curve

●Receiver Operating Characteristic (ROC) curve



Receiver-Operating Characteristic curve

●ROC curve demonstrates several things:

- It shows the tradeoff between sensitivity and specificity (any increase in sensitivity will be accompanied by a decrease in specificity).
- The closer the curve follows the left border and then the top border of the ROC space, the more accurate the test.
- The closer the curve comes to the 45-degree diagonal of the ROC space, the less accurate the test.
- The slope of the tangent line at a cut-point gives the likelihood ratio (LR) for that value of the test. You can check this out on the graph above.
- The area under the curve is a measure of test accuracy.

Continue with …

●CLT – EstimatesSamplingDistrX_bar.ppt

●ChiSquare_Tests.ppt

●MultipleLinearRegression.ppt

●Problems: C:\Ivo.dir\UCLA_Classes\2003\Winter\FOR6530\TestBank_StatPsych_files\ch22_NonParamTests.doc

Chi-Square χ^2 Test & Applications

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Chi-Square χ^2 Test & Applications

- Nature of the chi-square distribution: $X^2 = Y_1^2 + Y_2^2 + \dots Y_n^2$; where $Y_k \sim N(0,1)$
- Apply the chi-square distribution to:
  - Goodness-of-fit tests
  - Tests of independence between 2 variables
  - Tests comparing proportions from multiple populations
  - Tests of a single population variance.

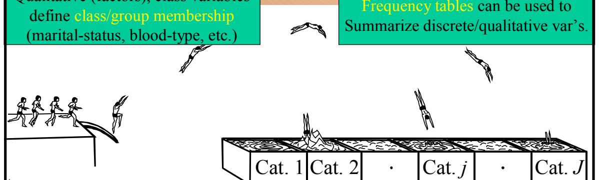
Key Terms

- Observed versus expected frequencies
- Number of parameters (for the target model distribution) to be estimated, *m*
- Number of categories used, *k*
- Contingency table
- Independent variables

1-dimensional tables – classify *n*-individuals in *J*-categories

Qualitative (factors), class variables define class/group membership (marital-status, blood-type, etc.)

Frequency tables can be used to Summarize discrete/qualitative var's.



tegory ...

Probability	p_1	p_2	·	·	·	p_j	·	·	·	p_J
Observed count	O_1	O_2	·	·	·	O_j	·	·	·	O_J
Expected count	E_1	E_2	·	·	·	E_j	·	·	·	E_J

$E_j = n\,p_j$

Goodness-of-Fit Tests

- The Question:
  - Does the distribution of sample data resemble a specified probability distribution, such as:
    - Binomial, HyperGeom, or Poisson discrete distributions.
    - Uniform, Normal, or Exponential continuous distributions.
    - Another probability distribution (perhaps data-driven)?
- Hypotheses: $\sum p_i = 1$
  - **H_0:** p_i = values expected **H_1**: $p_i \neq$ values expected

Goodness-of-Fit Tests

- **Rejection Region:**
  - Degrees of Freedom = $k - 1 - m$
    - where k = # of categories, m = # of parameters
    - **Uniform Discrete:** $m = 0$ so $df = k - 1$
    - **Binomial:** $m = 0$ when **p** is known, so $df = k - 1$; $m = 1$ when **p** is unknown, so $df = k - 2$
    - **Poisson:** $m = 1$ since μ is estimated, $df = k - 2$
    - **Normal:** $m = 2$ when μ and σ estimated, $df = k - 3$
    - **Exponential:** $m = 1$ since μ is estimated, $df = k - 2$

Goodness-of-Fit Tests

- Test Statistic:

$$\chi^2 = \sum_{j=1}^{k} \frac{(O_j - E_j)^2}{E_j}$$

where

O_j = Actual number **observed** in each class

E_j = Expected number, $p_j \cdot n$

Expected cell count = Total × specified cell probability

Goodness-of-Fit: An Example

- **Problem**: In a study of vehicle ownership, it has been found that 13.5% of U.S. households do not own a vehicle, with 33.7% owning 1 vehicle, 33.5% owning 2 vehicles, and 19.3% owning 3 or more vehicles. The data for a random sample of 100 households in a resort community are summarized below. At the 0.05 level of significance, can we reject the possibility that the vehicle-ownership distribution in this community differs from that of the nation as a whole?

# Vehicles Owned	# Households
0	20
1	35
2	23
3 or more	22

Goodness-of-Fit: An Example

# Vehicles	O_i	E_i	$[O_i - E_i]^2 / E_i$
0	20	13.5	3.1296
1	35	33.7	0.0501
2	23	33.5	3.2910
3+	22	19.3	0.3777
		Sum =	6.8484

I. **H_0:** $p_0 = 0.135$, $p_1 = 0.337$, $p_2 = 0.335$, $p_{3+} = 0.193$

Vehicle-ownership distribution in this community is the same as it is in the nation as a whole.

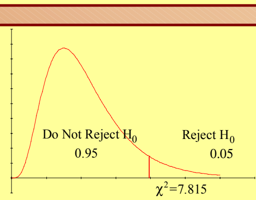
H_1: At least one of the proportions does not equal the stated value. Vehicle-ownership distribution in this community is not the same as it is in the nation as a whole.

Goodness-of-Fit: An Example

II. Rejection Region:

$\alpha = 0.05$

$df = k - 1 - m = 4 - 1 - 0 = 3$



III. Test Statistic:

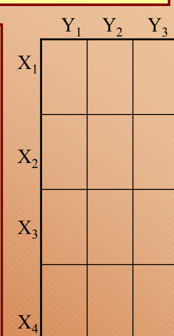
$\chi^2 = 6.8484$

IV. Conclusion: Since the test statistic of $\chi^2 = 6.8484$ falls below the critical value of $\chi^2 = 7.815$, we do not reject H_0 with at least 95% confidence.

V. Implications: There is not enough evidence to show that vehicle ownership in this community differs from that in the nation as a whole.

χ^2 Tests of Independence Between 2 Variables

- The Question:
  - Are the two variables independent? If the two variables of interest are independent ➔
    - The distribution of elements of one variable across the various levels does not affect how elements are distributed across the levels of the other variable.
    - the probability of an element falling in any level of the second variable is unaffected by knowing its level on the first dimension.



An Integrated Definition of Independence

- From basic probability:

  If two events are **independent**

  $P(A \text{ and } B) = P(A) \bullet P(B)$

  χ^2 Test of Independence:

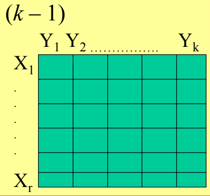
  If two variables are **independent**

  $P(row_i \text{ and } column_j) = P(row_i) \bullet P(column_j)$

Chi-Square Tests of Independence

- Hypotheses:
  - **H_0:** The two variables are independent.
  - **H_1**: The two variables are not independent.
- Rejection Region:
  - Degrees of freedom = $(r - 1)(k - 1)$
- Test Statistic:

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{k} \frac{(O_{i,j} - E_{i,j})^2}{E_{i,j}}$$



Chi-Square Tests of Independence

- Calculating expected values

$$E_{ij} = P(row_i \text{ and } column_j)\cdot n = P(row_i)\cdot P(column_j)\cdot n$$

$$= \frac{\#\text{elements in row}_i}{n}\cdot\frac{\#\text{elements in column}_j}{n}\cdot n$$

Canceling two factors of n,

$$E_{ij} = \frac{(\#\text{elements in row}_i)\cdot(\#\text{elements in column}_j)}{n}$$

Chi-Square Tests of Independence

An Example, Researchers in a California community have asked a sample of 175 automobile owners to select their favorite from three popular automotive magazines. Of the 111 import owners in the sample, 54 selected *Car and Driver*, 25 selected *Motor Trend*, and 32 selected *Road & Track*.

Of the 64 domestic-make owners in the sample, 19 selected *Car and Driver*, 22 selected *Motor Trend*, and 23 selected *Road & Track*. At the 0.05 level, is import/domestic ownership independent of magazine preference? What is the most accurate statement that can be made about the *p*-value for the test?

Chi-Square Tests of Independence

- First, arrange the data in a table.

	Car and Driver (1)	*Motor Trend (2)*	*Road & Track (3)*	**Totals**
Import (Imp)	54	25	32	**111**
Domestic (Dom)	19	22	23	**64**
Totals	**73**	**47**	**55**	**175**

- Second, compute the expected values and contributions to χ^2 for each of the six cells.
- Then to the hypothesis test ....

Chi-Square Tests of Independence

		Car and Driver (1)	*Motor Trend (2)*	*Road & Track (3)*
Import (Imp):	O -	54	25	32
	E -	46.3029	29.8114	34.8857
	χ^2 contribution -	1.2795	0.7765	0.2387
Domestic (Dom) :	O -	19	22	23
	E -	26.6971	17.1886	20.1143
	χ^2 contribution -	2.2192	1.3468	0.4140

$\Sigma\chi^2$ contributions = 6.2747

Chi-Square Tests of Independence

- I. Hypotheses:
  - H_0: Type of magazine and auto ownership are independent.
  - H_1: Type of magazine and auto ownership are not independent.
- II. Rejection Region:

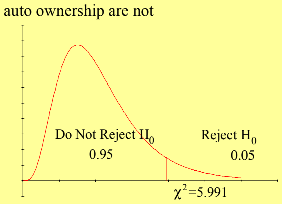
$\alpha = 0.05$

$df = (r - 1)(k - 1)$

$= (2 - 1)\bullet(3 - 1)$

$= 1 \bullet 2 = 2$

If $\chi^2 > 5.991$, reject H_0.



Chi-Square Tests of Independence

- III. Test Statistic:

$\chi^2 = 6.2747$

- IV. Conclusion:

Since the test statistic of 6.2747 falls beyond the critical value of 5.991, we reject the null hypothesis with at least 95% confidence.

- V. Implications:

There is enough evidence to show that magazine preference is not independent from import/domestic auto ownership.

- *p*-value: In a cell on a Microsoft Excel spreadsheet, type: =CHIDIST(6.2747,2). The answer is: ***p*-value = 0.043398**