Homework 6 Solutions

Problem 1: (6 points, 1 point for a problem)

1) $N(5000, 112^2)$

2) $N(5000, 4*28^2) = N(5000, 56^2)$

3) they differ in SD: plan 2 has smaller SD (a half of that of plan 1)

4) $z = \frac{5100 - 5000}{112} = 0.8928$

Prob(spend more than budget)=prob(Z>z)=1-0.814=0.186

5) $z = \frac{5100 - 5000}{56} = 1.7857$

Prob(spend more than budget)=prob(Z>z)=1-0.963=0.037

6) plan 2

Problem 2: (7 points total)

Let $X_{ij}, i = 1,2,3,4,5, j = 1,2,$ denote the output of i-th die in j-th throw. Then clearly $Y = \sum_i \sum_j X_{ij}$. And we can also see that X_{ij}'s are independent of each other and identically distributed.

$E(X_{11}) = \frac{1}{8} \sum_i i = 4.5$ and $\text{var}(X_{11}) = \sum_i \frac{1}{8}(i - 4.5)^2 = \frac{21}{4}$

hence $\mu_Y = 4.5 * 10 = 45$, (2 points)

and $\sigma_Y = \sqrt{\text{var}(Y)} = \sqrt{10 * \frac{21}{4}} = 7.25$ (2 points)

Approximately, $\bar{Y}$ will follow a Normal distribution by Central Limit theorem. (1 point)

And $E(\bar{Y}) = E(Y_1) = 45$, (1 point)

$sd(\bar{Y}) = \frac{1}{\sqrt{n}} sd(Y_1) = 7.25 / 3 = 2.42$ (1 point)

Problem 3: (6 points total)

1) For treatment group: mean=14.1
Sd=2.47
For control group: mean=183/19=9.63
Sd=3.34 (2 points total: 1 point for each group)

2) $sd(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{sd_1^2}{n_1} + \frac{sd_2^2}{n_2}} = \sqrt{\frac{2.47^2}{20} + \frac{3.34^2}{19}} = 0.944$

hence a 95% CI for the difference in mean is ($t_{\alpha=0.025,\ 19-1} =$ 2.101)

$[14.1 - 9.63 - 2.101 * 0.944 \quad , \quad 14.1 - 9.63 + 2.101 * 0.944] = [2.49 \quad , \quad 6.45]$ (1 point) because both end are positive, we can fairly safely say that the mean of treatment group is larger than the control group by a number between 2.49 and 6.45 (1 point)

3) (19+20)*4=156 (1 point)

4)

It may contain the true mean, but there is possibility that it doesn’t contain the true mean. If we do such experiments for many times, about 95% of the CI’s constructed in the same way will contain the true mean. (but we don’t know that for a specific one.) (1 point)