

STAT 110 A, Probability & Statistics for Engineers I

UCLA Statistics, Spring 2004

http://www.stat.ucla.edu/~dinov/courses_students.html

SOLUTION HOMEWORK 1

Due Date: Friday, Apr. 16, 2004

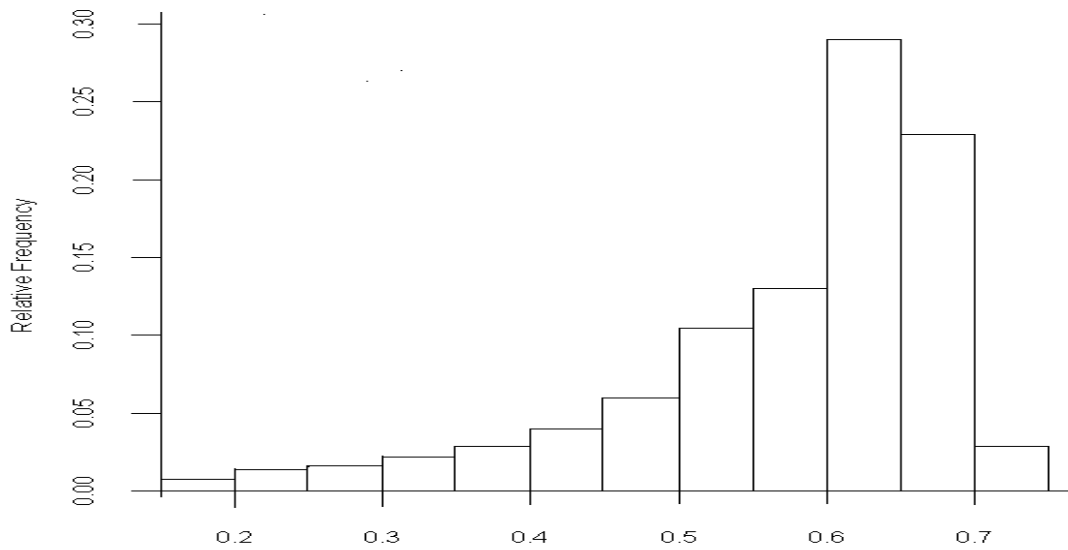
http://www.stat.ucla.edu/%7Edinov/courses_students.dir/04/Spring/Stat110A.dir/HWs.dir/HW1.html

Problem 1 (There is a total of 100 points for this assignment.)

a) (2 points for relative frequency; 2 points for cumulative relative frequency)

| Class interval for the clearness index | Number of days | Relative Frequency | Cumulative Relative Frequency |
|--|----------------|--------------------|-------------------------------|
| 0.16 - 0.20 | 3 | 0.008219 | 0.00822 |
| 0.21 - 0.25 | 5 | 0.013699 | 0.02192 |
| 0.26 - 0.30 | 6 | 0.016438 | 0.03836 |
| 0.31 - 0.35 | 8 | 0.021918 | 0.06027 |
| 0.36 - 0.40 | 12 | 0.032877 | 0.09315 |
| 0.41 - 0.45 | 16 | 0.043836 | 0.13699 |
| 0.46 - 0.50 | 24 | 0.065753 | 0.20274 |
| 0.51 - 0.55 | 39 | 0.106849 | 0.30959 |
| 0.56 - 0.60 | 51 | 0.139726 | 0.44932 |
| 0.61 - 0.65 | 106 | 0.290411 | 0.73973 |
| 0.66 - 0.70 | 84 | 0.230137 | 0.96986 |
| 0.71 - 0.75 | 11 | 0.030137 | 1.00000 |

b) The histogram is negatively-skewed. (2 points for explanation; 2 points for histogram)



c) The proportion of days with a clearness index smaller than 0.35 is
 $(3+5+6+8)/365 = 0.06027$ (2 points)

d) The proportion of days with a clearness index of at least 0.65 is
 $(84+11)/365 = 0.26$ (2 points)

Problem 2

a) 32|55 (4 points)

33|49

34|

35|6699

36|34469

37|03345

38|9

39|2347

40|23

41|

42|4

The display is reasonably symmetric, so the mean and median will be close. (2 points)

b) The sample mean is $9638/26 = 370.7$ (2 points)

The sample median is $(369+370)/2 = 369.50$ (2 points)

c) The largest value (424) could be increased by any amount. Doing so will not change the fact that middle two observations are 369 and 370, and hence, the median will not change. (2 points)

d) The sample mean expressed in minutes is $(370.7\text{sec})/(60\text{sec}) = 6.18$ min; the sample median is $(369.50\text{sec})/(60\text{sec}) = 6.16$ min. (2 points for mean; 2 points for median)

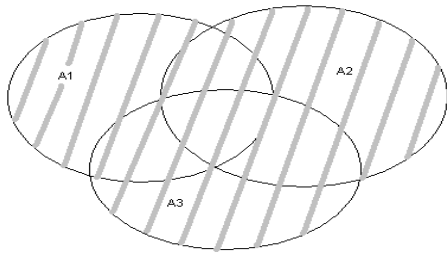
Problem 3

a) The sample mean is $(14.5+25.6+52.4+66.3+69.3+69.8+76.2)/7 = 53.44$; the sample median is 66.3. The fact that mean is smaller than median suggests that observations on the lower end of the sample are more stretched out than those on the upper end. (2 points for sample mean; 2 points for sample median; 2 points for explanation)

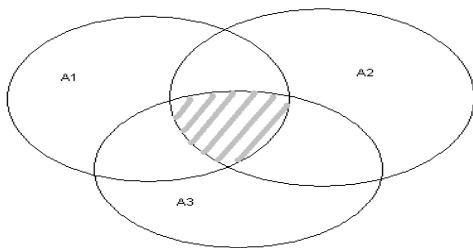
b) The sample variance is $[23448.03-(374.1)^2/7]/6 = 582.51$; the standard deviation is $\sqrt{582.51} = 24.14$ (2 points for sample variance; 2 points for standard deviation)

Problem 4

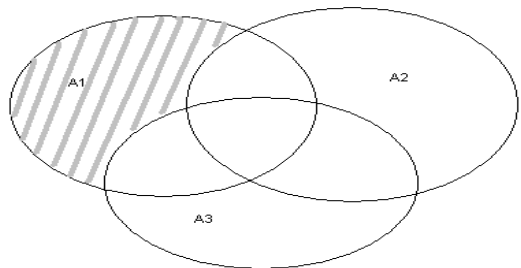
a) $A1 \cup A2 \cup A3$ (2 points for equation; 2 points for Venn Diagram)



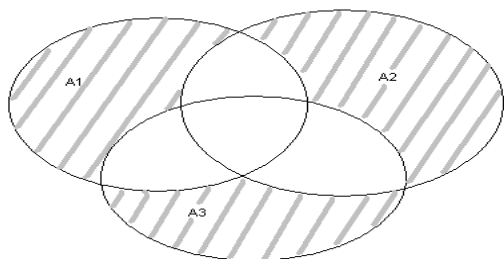
b) $A1 \cap A2 \cap A3$ (2 points for equation; 2 points for Venn Diagram)



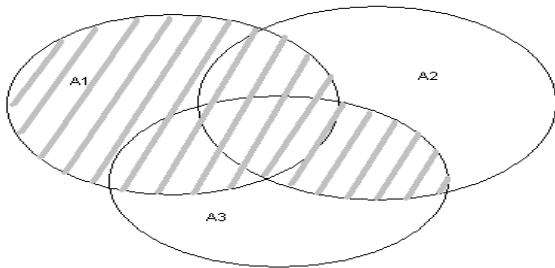
c) $A1 \cap A2' \cap A3'$ (2 points for equation; 2 points for Venn Diagram)



d) $(A1 \cap A2' \cap A3') \cup (A1' \cap A2 \cap A3') \cup (A1' \cap A2' \cap A3)$
(2 points for equation; 2 points for Venn Diagram)



e) $A1 \cup (A2 \cap A3)$ (2 points for equation; 2 points for Venn Diagram)



Problem 5

a) $P(A1 \cup A2) = P(A1) + P(A2) - P(A1 \cap A2) = 0.22 + 0.25 - 0.11 = 0.36$ (2 points)

b) $P(A1' \cap A2') = 1 - P(A1 \cup A2) = 1 - 0.36 = 0.64$ (2 points)

c) $P(A1 \cup A2 \cup A3) = P(A1) + P(A2) + P(A3) - P(A1 \cap A2) - P(A1 \cap A3) - P(A2 \cap A3) + P(A1 \cap A2 \cap A3)$
 $= 0.22 + 0.25 + 0.28 - 0.11 - 0.05 - 0.07 + 0.01 = 0.53$ (2 points)

d) $P(A1' \cap A2' \cap A3') = 1 - P(A1 \cup A2 \cup A3) = 1 - 0.53 = 0.47$ (2 points)

e) $P(A1' \cap A2' \cap A3) = P(A3) - P(A1 \cap A3) - P(A2 \cap A3) + P(A1 \cap A2 \cap A3)$
 $= 0.28 - 0.05 - 0.07 + 0.01 = 0.17$ (2 points)

f) $P[(A1' \cap A2') \cup A3] = 0.47 + 0.28 = 0.75$ (2 points)

Problem 6

a) $P(M,H) = 0.10$ (2 points)

b) $P(\text{low auto}) = P[(L,N),(L,L),(L,M),(L,H)] = 0.04 + 0.06 + 0.05 + 0.03 = 0.18$ (2 points)
 $P(\text{low homeowner's}) = 0.06 + 0.10 + 0.03 = 0.19$ (2 points)

c) $P(\text{same deductible for both}) = P(L,L) + P(M,M) + P(H,H)$
 $= 0.06 + 0.20 + 0.15 = 0.41$ (2 points)

d) $P(\text{deductibles are different}) = 1 - P(\text{same deductible for both})$
 $= 1 - 0.41 = 0.59$ (2 points)

e) $P(\text{at least one low}) = P(L,N) + P(L,L) + P(L,M) + P(L,H) + P(M,L) + P(H,L)$
 $= 0.04 + 0.06 + 0.05 + 0.03 + 0.10 + 0.03 = 0.31$ (2 points)

f) $P(\text{neither low}) = 1 - P(\text{at least one low}) = 1 - 0.31 = 0.69$ (2 points)

Problem 7

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.70 + 0.80 - 0.85 = 0.65$$

$$P(B \cap C) = P(B) + P(C) - P(B \cup C) = 0.80 + 0.75 - 0.95 = 0.60$$

$$P(A \cap C) = P(A) + P(C) - P(A \cup C) = 0.70 + 0.75 - 0.90 = 0.55$$

$$\begin{aligned} P(A \cap B \cap C) &= P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(A \cap C) \\ &= 0.98 - 0.7 - 0.8 - 0.75 + 0.65 + 0.55 + 0.6 = 0.53 \end{aligned}$$

a) $P(A \cup B \cup C) = 0.98$ (2 points)

b) $1 - P(A \cup B \cup C) = 1 - 0.98 = 0.02$ (2 points)

c) $P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) = 0.75 - 0.55 - 0.60 + 0.53$
 $= 0.13$ (2 points)

d) $P(A \cup B \cup C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + 2 P(A \cap B \cap C)$
 $= 0.98 - 0.65 - 0.60 - 0.55 + 2(0.53) = 0.24$ (2 points)

Problem 8

a) $1 - P(A1) = 1 - 0.12 = 0.88$ (2 points)

b) $P(A1 \cap A2) = P(A1) + P(A2) - P(A1 \cup A2) = 0.12 + 0.07 - 0.13 = 0.06$ (2 points)

c) $P(A1 \cap A2) - P(A1 \cap A2 \cap A3) = 0.06 - 0.01 = 0.05$ (2 points)

d) $1 - P(A1 \cap A2 \cap A3) = 1 - 0.01 = 0.99$ (2 points)