Stat13 Homework 4

http://www.stat.ucla.edu/~dinov/courses_students.html

Suggested Solutions

7. The shell of the land snail Linocolaria marfensiana has two possible color forms: streaked and pallid. In a certain population of these snails, 60% of the individuals have streaked shells. Suppose a random sample of ten snails is to be chosen from the population: let p hat be the sample proportion of streaked snails.

This problem is binomial with n = 10 and p = .6

7a. P(p hat = .5) = (252)($.6^5$)($.4^5$) = .2007
7b. P(p hat = .6) = (210)($.6^6$)($.4^4$) = .2508
7c. P(p hat = .7) = (120)($.6^7$)($.4^3$) = .2150
7d. P(.5 =< p_hat =< .7) = .2007 + .2508 + .2150 = .6665
7e. The percentage of samples in which p hat is within +/- .1 of p = .6665 (same as in part 7d)

18. The heights of a certain population of corn plants follows a normal distribution with mean 145 cm and standard deviation 22 cm.

18a. What percentage of the plants are between 135 and 155 cm tall?
Z = (155-145)/22 = .45 and the corresponding area is .6736
Z = (135-145)/22 = -.45 and the corresponding area is .3264
So .6736 - .3264 = .3472, or 34.72% of the plants.

18b. Suppose we were to choose at random from the population a large number of samples of 16 plants each. In what percentage of the samples would the sample mean height be between 135 cm and 155 cm?
So n = 16 and sigma y bar = 22/sqrt(16) = 5.5
Z = (155-145)/5.5 = 1.82 and the corresponding area is .9656
Z = (135-145)/5.5 = -1.82 and the corresponding area is .0344
So .9656 - .0344 = .9312, or 93.12% of the plants.

18c. If Y bar represents the mean height of a random sample of 16 plants from the population, what is P(135=< Y bar =< 155)?
.9312 (from part b)

18d. If Y bar represents the mean height of a random sample of 36 plants from the population, what is P(135=< Y bar =< 155)?
So n = 36 making sigma y bar = 22/sqrt(36) = 3.67
Z = (155-145)/3.67 = 2.72 and the corresponding area is .9967
Z = (135-145)/3.67 = -2.72 and the corresponding area is .0033
So .9967 - .0033 = .9934, or 99.34% of the plants.

33. In the United States, 44% of the population has type O blood. Suppose a random sample of 12 persons is taken. Find the probability that 6 of the persons will have type O blood (and 6 will not).

33a. Using the **binomial** distribution formula:
We will let type O blood = success so j = 6. We have n = 12 and p = .44
P(type O blood) = $(924)(.44^6)(.56^6) = .2068$

33b. Using the normal approximation with continuity correction
mean = (n)(p) = (12)(.44) = 5.28
standard deviation = sqrt[(n)(p)(1-p)] = sqrt[(12)(.44)(.56)] = 1.72
So $P(X = 6) = P(5.5 < X < 6.5)$
Z = (5.5-5.28)/1.72 = .13 and the corresponding area is .5517
Z = (6.5-5.28)/1.72 = .71 and the corresponding area is .7580
So .7580 - .5517 = .2063

49. Consider taking a random sample of size 25 from a population in which 42% of the people have type A blood. What is the probability that the sample proportion with type A blood will be greater than .44? Use the normal approximation to the binomial with continuity correction.

For the normal approximation to the sampling distribution of p hat, the mean p = .42 and the standard deviation is sqrt[{(p)(1-p)}/n] = sqrt[{(.42)(.58)}/25] = .0987
Continuity correction: (1/2)(1/25)= .02
Z = (.46-.42)/.0987 =.405 and the corresponding area is 1 - .6590 = .3410.