[Stat13 Homework 7](http://www.stat.ucla.edu/~dinov/courses_students.html)

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Suggested Solutions

Question 7.50

Let 1 denote infected and 2 denote noninfected.

H_0: Malaria does not affect red cell count ($\mu_1=\mu_2$)

H_A: Malaria reduces red cell count ($\mu_1<\mu_2$)

We note that $\bar{y}_1 > \bar{y}_2$, so the data do not deviate from H_0 in the direction specified by H_A. Thus, P>0.50

Part (a):

H_0 is not rejected. There is no evidence that malaria reduces red cell count in this population.

Part (b):

Same as part (a).

Note: If H_A is reversed ($\mu_1>\mu_2$), then H_0 would be rejected at α=0.10. Thus, this exercise illustrates the importance of the directionality check.

Question 7.51

Let 1 denote experiment (to be hypnotized) and 2 denote control.

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{0.621^2}{8}+\frac{0.652^2}{8}} = 0.3183$$

$$t_s = \frac{6.169-5.291}{0.3183} = 2.76$$

With $df = n_1 + n_2 - 2 = 14$, Table 4 gives $t_{0.01} = 2.624$ and $t_{0.005} = 2.977$.

Part (a):

H_0: Mean ventilation is the same in the "to be hypnotized" condition than in the "control" condition ($\mu_1=\mu_2$).

H_A: Mean ventilation is different in the "to be hypnotized" condition than in the "control" condition ($\mu_1\neq\mu_2$)

H_0 is rejected. There is sufficient evidence ($0.01<P<0.02$) to conclude that mean ventilation is higher in the "to be hypnotized" condition than in the "control" condition.

Part (b):

H$_0$: Mean ventilation is the same in the "to be hypnotized" condition than in the "control" condition ($\mu_1=\mu_2$).

H$_A$: Mean ventilation is higher in the "to be hypnotized" condition than in the "control" condition ($\mu_1>\mu_2$)

H$_0$ is rejected. There is sufficient evidence ($0.005<P<0.01$) to conclude that mean ventilation is higher in the "to be hypnotized" condition than in the "control" condition.

Part (c):
The nondirectional alternative (part (a)) is more appropriate. According to the narrative, the researchers formulated the directional alternative in part (b) after they had seen the data. Thus, it would not be legitimate for them to use a directional alternative.

Question 7.52
Let 1 denote experiment (to be hypnotized) and 2 denote control (α=0.1).

H$_0$: Extra nitrogen does not enhance plant growth ($\mu_1=\mu_2$).

H$_A$: Extra nitrogen does enhance plant growth ($\mu_1<\mu_2$).

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{0.54^2}{5}+\frac{0.67^2}{5}} = 0.3848$$

$$t_s = \frac{3.62-4.17}{0.3848} = -1.43$$

With $df = n_1 + n_2 - 2 = 8$, Table 4 gives $t_{0.10} = 1.397$ and $t_{0.05} = 1.860$.

H$_0$ is rejected. There is some evidence ($0.05<P<0.10$) to conclude that extra nitrogen enhances plant growth under these conditions.

Question 7.55
The bound of p-value is: 0.03<p-value<0.05.
Thus, we would reject H$_0$ and conclude that ancy does, indeed, inhibit growth.

Question 7.57
The proponents are confused. They are speaking as if it is known that $\mu_1-\mu_2$=4lb/acre, whereas the field trial indicates only that $\bar{y}_1-\bar{y}_2=4$ lb/acre. That statistician's data analysis indicates that the trial gives only weak information about $\mu_1-\mu_2$; in fact, the results do not even show whether $\mu_1-\mu_2$ is positive, let alone that it is equal to 4lb/acre.

Question 7.59
Let 1 denote male and 2 denote female.

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{0.62^2 + 0.53^2} = 0.8157$$

$$CI = (137.21-137.18) \pm (1.977)(0.8157) = (-1.6, 1.6)\text{, using df=140}$$

We can be 95% confident that the mean difference does not exceed 1.6 beats per minute, which is small and unimportant (in comparison with, for example, ordinary fluctuations in heart rate from one minute to the next.)

Question 7.65

$$\frac{|\mu_1-\mu_2|}{\sigma} = \frac{44.4}{69.6} = 0.64$$

Part (a):
Table 5 gives n=39.
Part (b):
Table 5 gives n=30.

Question 7.80

Let 1 denote experimental (to be hypnotized) and 2 denote control.
Part (a):

H_0: Ventilation is not differently affected by the "to be hypnotized" and the "control" conditions.

H_A: Ventilation is differently affected by the "to be hypnotized" and the "control" conditions.

$$K_1 = 53, K_2 = 11, U_S = 53$$

With n=8 and n'=8, Table 6 gives $0.02<P<0.05$. H_0 is rejected. There is sufficient evidence ($0.02<P<0.05$) to conclude that ventilation rate tends to be higher under the "to be hypnotized" condition than under the "control" condition.

Question 7.81

Part (a):
U_S=9. With n=n'=3, U_S=9 is under the 0.10 heading and is the largest entry listed. Thus, $0.05<P<0.10$.
Part (b):
U_S=16. With n=n'=4, U_S=16 is under the 0.05 heading and is the largest entry listed. Thus, $0.02<P<0.05$.
Part (a):
U_S=25. With n=n'=5, U_S=25 is under the 0.01 heading and is the largest entry listed. Thus, $0.002<P<0.01$.

Question 7.82

Part (a):

H_0: There is no sex difference in preening behavior.

H_A: There is a sex difference in preening behavior.

From n=n'=15, the largest critical value is 189, which is under the 0.001 heading for a nondirectional alternative. It follows $P<0.001$, so H_0 is rejected. There is sufficient evidence ($P<0.001$) to conclude that females tend to preen longer than males.

Part (b):

H_0: There is no sex difference in preening behavior. ($\mu_1=\mu_2$)

H_A: There is a sex difference in preening behavior. ($\mu_1\neq\mu_2$)

$$t_s = \frac{2.127 - 4.093}{0.7933} = -2.48$$

With $df = n_1 + n_2 - 2 = 28$, Table 4 gives $t_{0.01} = 2.467$ and $t_{0.005} = 2.763$, so that

$0.01<P<0.02$. Formula (7.1) yields df=15.1 and the conservative approach of df = min{n_1-1,n_2-1} gives df=14. For either of these df values we get $0.02<P<0.04$. In any case, H_0 is not rejected, since $P>0.01$. There is sufficient evidence to conclude that there is a sex difference in preening behavior.

Part (c):

Both tests require independent, random samples. The condition required for the t-test but not for the Wilcoxon-Mann-Whitney test is that the population distributions are normal. The frequency distribution for the females is highly skewed, due to the two large observations of 10.7 and 11.7. This casts doubt on the normality condition.

Part (d):

$$K_1 = 0+0+0+0+0+0.5+1+1.5+1.5+2+2+3.5+5+8.5+10 = 35.5$$

$$K_2 = 5.5+8+3(11.5)+3(13)+13.5+14+5(15) = 189.5$$

where 1 denotes male and 2 denotes female.

Question 7.83

Part (a):

Let 1 denote singly housed and let 2 denote group-housed.

H_0: There is no difference in benzo(a)pyrene concentrations between singly housed and group-housed mice.

H_A: Benzo(a)pyrene concentrations tend to be higher in group-housed mice than in singly housed mice.

$$K_1 = 0,\ K_2 = 25,\ U_S = 25$$

and the shift in the data is in the direction predicted by H_A. With n=n'=5, U_S=25 is under the 0.005 heading for a directional alternative and is the largest entry listed. Thus,

$0.001<P<0.005$ and H_0 is rejected. There is sufficient evidence ($0.001<P<0.005$) to conclude that benzo(a) pyrene concentration tend to be higher in group-housed mice than in singly housed mice.

Part (b):

A directional alternative is valid in this case because the researchers were investigating the hypothesis that licking or biting other mice lends to increase benzo(a)-pyrene concentration. If access to other mice affects benzo(a)pyrene concentration, the effect would be to increase the concentration; a decrease is not plausible.

Question 7.84

Let 1 denote joggers and let 2 denote fitness program entrants.

H_0: There is no difference in resting blood concentration of HBE between joggers fitness program entrants.

H_A: There is difference in resting blood concentration of HBE between joggers fitness program entrants.

$$K_1 = 93.5,\ K_2 = 71.5,\ U_S = 93.5$$

With n=15 and n'=11, 108 is under the 0.20 heading for a nondirectional alternative and is the smallest entry listed. Thus, $P>0.20$ and H_0 is not rejected. There is insufficient evidence ($P>0.20$) to conclude that there is a difference in resting blood concentration of HBE between joggers fitness program entrants.

Question 7.89

Part (a):

H_0: Mechanical milking does not produce different cell count than manual milking ($\mu_1=\mu_2$).

H_A: Mechanical milking produces higher cell count than manual milking ($\mu_1>\mu_2$)

$$t_s = \frac{1215.6-219.0}{427.54} = 2.33$$

With df=18, Table 4 gives $t_{0.02} = 2.214$ and $t_{0.01} = 2.552$. Formula (7.1) yields df=9.2≈9; with df=9, Table 4 gives $t_{0.025} = 2.262$ and $t_{0.02} = 2.398$. Using either df value, $P<0.05$ and H_0 is rejected. There is sufficient evidence to conclude that mechanical milking produces higher cell count than manual milking. (The data support the investigator's suspicion)

Part (b):

H_0: Mechanical milking does not produce different cell count than manual Milking.

H_A: Mechanical milking produces higher cell count than manual milking.

$U_S=69$. The shift in the data is in the direction predicted by H_A. With n=n'=10, the entry under 0.10 for a directional alternative is 68 and the entry under 0.05 for a directional alternative is 73. Thus, we do not reject H_0. There is insufficient evidence (0.05<P<0.10) to conclude that mechanical milking produces higher cell count than manual milking (The data do not support the investigator's suspicion). [Note that this contradicts the conclusion from part (a)].

Part (c):

Both tests require independent, random samples. The condition required for the t-test but not for the Wilcoxon-Mann-Whitney test is that the population distributions are normal. The frequency distribution for the mechanical group is highly skewed, with some observations (2996 and 3452) that are much greater than the others. This casts doubt on the normality condition.

Part (d):

$$K_1 = 10+7+0+10+10+7+1+10+4+10 = 69$$
$$K_2 = 3+2+1+2+2+3+5+5+5+3 = 31$$

where 1 denotes mechanical and 2 denotes manual.

Question 7.93

Let 1 denote Vermilion River and let 2 denote Black River.

H_0: The populations from which the two samples were drawn have the same distribution of tree species per plot.

H_A: Biodiversity is greater along the Vermilion River than along the Black River.

$$K_1 = 1.5, K_2 = 106.5, U_S = 80$$

The data deviate from H_0 is the direction specified by H_A. With n=n'=13 and a directional alternative, the 0.10 entry in Table 6 is 79 and the 0.05 entry is 84. Thus, the p-value is between 0.05 and 0.10, so we reject H_0. There is sufficient evidence (0.05<P<0.10) to conclude that biodiversity is greater along the Vermilion River than along the Black River.

Question 7.95

H_0: Ovarian pH is not related to progesterone response ($\mu_1=\mu_2$).

H_A: Overian pH is related to progesterone response ($\mu_1\neq\mu_2$).

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{0.129^2}{18}+\frac{0.081^2}{6}} = 0.04492$$

$$t_s = \frac{7.373-7.708}{0.04492} = -7.46$$

With df=22, Table give $t_{0.0005} = 3.792$. Thus, P<0.001, so we reject H_0. There is sufficient evidence (P<0.001) to conclude that ovarian pH is lower among responders to propesterone than among nonresponders.

Question 7.99

Let 1 denote low chromium and let 2 denote normal.

H_0: Low chromium diet does not affect GITH ($\mu_1=\mu_2$).

H_A: Low chromium diet does affect GITH ($\mu_1 \neq \mu_2$).

$$\bar{y}_1 = 51.75 \quad s_1 = 5.526$$
$$\bar{y}_2 = 53.17 \quad s_2 = 4.123$$
$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{5.526^2}{14}+\frac{4.123^2}{10}} = 1.970$$
$$t_s = \frac{51.75-53.17}{1.970} = -0.72$$

Formula (7.1) gives df=21.9; $t(22)_{0.20} = 0.858$, so P>0.40. Using a computer, we get P=0.48. Thus, we do not reject H_0. There is insufficient evidence (P=0.48) to conclude that low chromium diet affects GITH in rats.

Question 7.101

Part (a):

$$CI = (51.75-53.17) \pm (2.074)(1.970) = (-5.5, 2.7)\text{, using df=22}$$

Part (b):

All values in the confidence interval are smaller in magnitude than 8000 cpm/gm; thus the data support the conclusion that the difference is "unimportant".

Part (c):

The confidence interval indicates that the difference could be larger in magnitude than 4000 cpm/gm or smaller; thus the data do not indicate whether the difference is "unimportant".

Question 7.104

Part (a):

False. The confidence interval include zero, so we are not confident that μ_1 and μ_2 are different.

Part (b):

True. This is what a confident interval tells us.

Part (c):

False. We know that $\bar{y}_1 - \bar{y}_2 = 6.9$.

Part (d):

False. The confidence interval is used to make an inference about the difference between μ_1 and μ_2; it does not tell us about individual data points (such as the length of hospitalization for a nitric oxide infant).