http://www.stat.ucla.edu/~dinov/courses_students.dir/10/Winter/STAT13.1.dir/

STAT 13 Homework 5 Solutions

1. Problem 1

P(p^ = 0) = 0.078

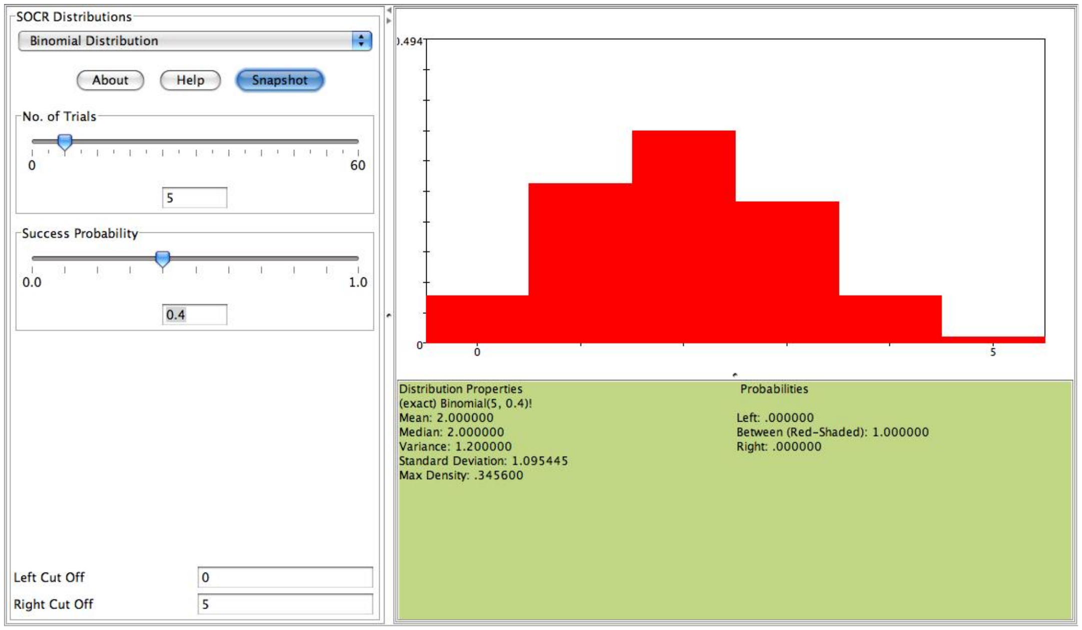
P(p^ = 0.2) = 0.259

P(p^ = 0.4) = 0.346

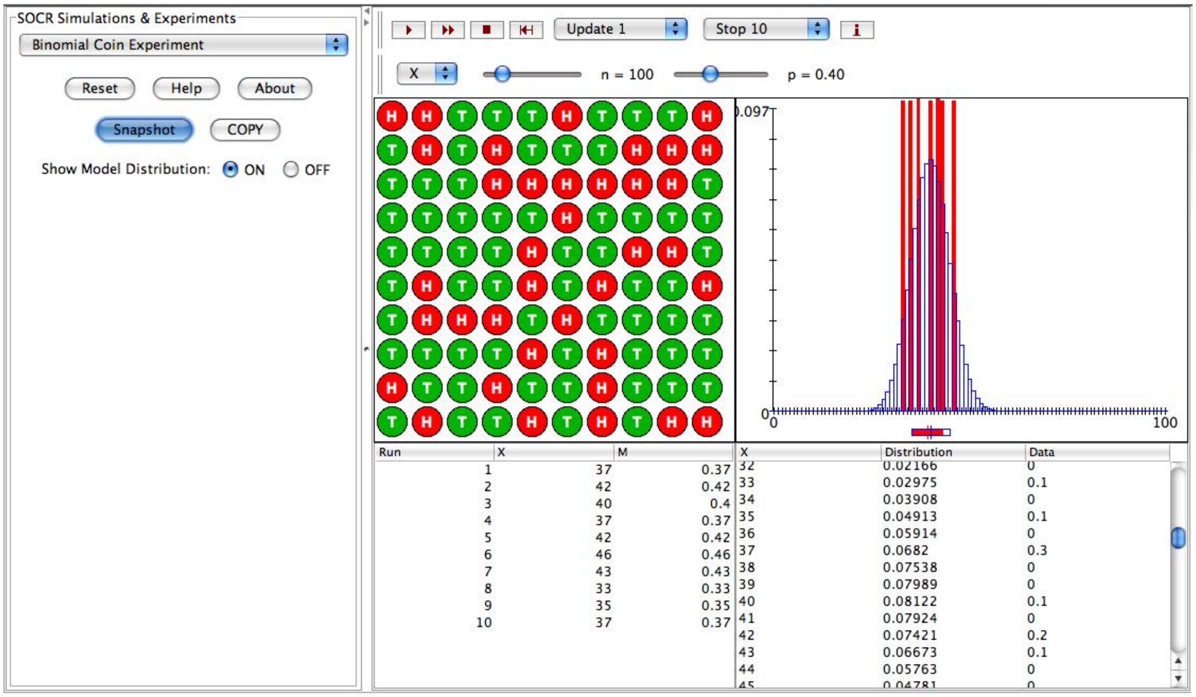
P(p^ = 0.6) = 0.23

P(p^ = 0.8) = 0.077

P(p^ = 1) = 0.01



The Binomial Coin Experiment shows that the empirical and theoretical probabilities don't really come close for most of the possible values. But this is to be expected, as we only ran 10 trials. The empirical probabilities won't really become closer to the theoretical ones if we increased the number of coins tossed in each experiment to 100, because we'd still only be running 10 trials. As is shown from the graph below, it’s not even close. However, the empirical and theoretical probabilities should get closer if we instead ran 100 experiments of tossing 20 coins at a time.



2. Problem 2

We can solve these by using the margin of error formula to first calculate the critical value z, and then use that to find $P(E) = P(-z_{\alpha/2} < Z < z_{\alpha/2})$.

n=15: $100 = z_{\alpha/2} * 400/\sqrt{15}$

$z_{\alpha/2} = 0.968$

P(E) = 0.667

n=60: $100 = z_{\alpha/2} * 400/\sqrt{60}$

$z_{\alpha/2} = 1.936$

P(E) = 0.947

So as the n gets larger, so does P(E).

3. Problem 3:

If p^ is closer to 1/2 than 9/16, that means it’s less than the midpoint, 17/32.

n=1: Technically, when n=1, either that one flower is purple or it isn't, so p^ is either 0 (not purple) or 1 (purple). Since there's a 9/16 chance it's purple, the probability of the misleading event is simply 7/16. Using the normal approximation anyway, however, we have mean = 9/16, s.d. = $\sqrt{(1*9/16*7/16)}$ = 0.496, and we're trying to find $P(X < 17/32)$.

$P(X < 17/32) = P(Z < -0.063) = 0.4749$.

n=64: We have mean = 64*9/16 = 36, s.d. = $\sqrt{(64*9/16*7/16)}$ = 3.97.

$P(X < 64*17/32) = P(X < 34) = P(Z < -0.504) = 0.3071$.

n=320: We have mean = 320*9/16 = 180, s.d. = $\sqrt{(320*9/16*7/16)}$ = 8.87.

$P(X < 320*17/32) = P(X < 170) = P(Z < -1.127) = 0.1299$.

4. Problem 4

Since we want the weight of 10 mice combined to be larger than 90g, that implies the *average* weight of those 10 mice had to be larger than 9.0g. We're looking at a sampling distribution where mean = 8.3g, s.d. = $1.7/\sqrt{10}$ = 0.5376.

P(X-bar > 9.0) = P(Z > 1.302) = 0.0964, so 9.64% of the litters will weigh 90g or more.

5. Problem 5

Because the sample size is only 6, we need to use the Student's T-distribution instead of the normal distribution. So we calculate:

$28.7 \pm 2.57*4.6/\sqrt{6} = 28.7 \pm 4.827 = [23.873, 33.527]$.

$$CI(\alpha): \overline{x} \pm t_{\frac{\alpha}{2}} E.$$

This means that we are 95% confident that the true population mean of blood serum concentrations of Gentamicin (in μg/mLi) should be somewhere between 23.873 and 33.527. It is NOT typical that the confidence interval contains almost all the observations, as the interval only deals with where the population mean μ is, and has nothing to do with the individual observations themselves. (Consider a bimodal distribution, where a good confidence interval should contain almost *none* of the observations.)

6. Problem 6

With a sample size of 10, we're still using the Student's T-distribution to help us calculate the confidence interval.

$13.0 \pm 2.262*12.4/\sqrt{10} = 13.0 \pm 8.87 = [4.13, 21.87]$.

This means that we are 95% confident that the true difference in HBE hormone levels (in pg/mLi) from January to May after an exercise program is somewhere between 4.13 and 21.87. If the true difference μ = 0, that implies there really was no change in people's HBE levels on average. But since zero is not contained in the interval, and these numbers measure the *decrease* in HBE levels, it seems very likely that the program *did* help decrease HBE levels.