UCLA STAT 10

Introduction to Statistical Reasoning

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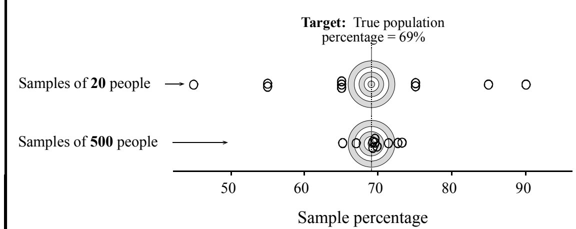
Chapter 6 –

Measurement Error, Chance and Uncertainty

Newtonial science vs. chaotic science

- Article by Robert May, Nature, vol. 411, June 21, 2001
  - Science we encounter at schools deals with crisp certainties (e.g., prediction of planetary orbits, the periodic table as a descriptor of all elements, equations describing area, volume, velocity, position, etc.)
  - As soon as uncertainty comes in the picture it shakes the foundation of the deterministic science, because only probabilistic statements can be made in describing a phenomenon (e.g., roulette wheels, chaotic dynamic weather predictions, Geiger counter, earthquakes, Others?)
  - What is then science all about – describing absolutely certain events and laws alone, or describing more general phenomena in terms of their behavior and chance of occurring? Or may be both!

Variation in sample percentages

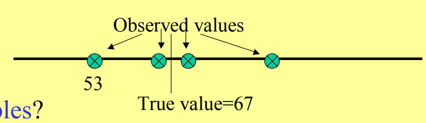


Comparing percentages from 10 different surveys each of 20 people with those from 10 surveys each of 500 people (all surveys from same population).

From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 2000.

Measurement Error

- No matter how carefully a measurement of a single unit is made it oftern comes out a bit different. Do repeated measurements to find out by how much different each observation is!
- The SD of a series of repeated measurements estimates the **likely size of the chance error** in a single measurement of the process being observed.



- Examples?

Random or chance error …

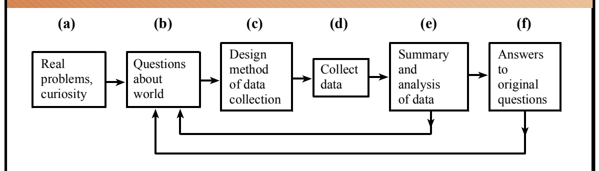
- Random or chance error is the difference between the sample-value and the true population-value (e.g., 53 vs. 67, in the above example).

The Subject of Statistics

Statistics is concerned with the process of finding out about the world and how it operates -

- in the face of variation and uncertainty
- by collecting and then making sense (interpreting, summarizing) of data.

The investigative process



Questions

- What are two ways in which random observations arise and give examples. (random sampling from finite population – randomized scientific experiment; random process producing data, observational data, surveys.)
- What is a parameter? Give two examples of parameters. (characteristic of the data – mean, 1st quartile, std.dev.)
- What is an estimate? How would you estimate the parameters you described in the previous question?
- What is the distinction between an estimate (p^ value calculated form obs'd data to approx. a parameter) and an estimator (P^ abstraction the the properties of the ransom process and the sample that produced the estimate) ? Why is this distinction necessary? (effects of sampling variation in P^)

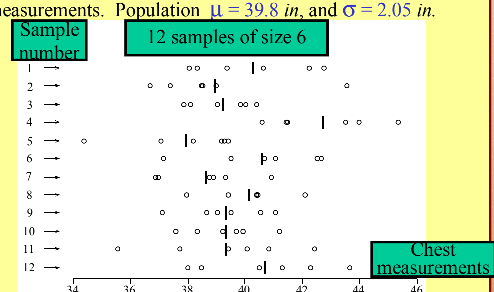
Review

- Let $\{x_{1,1}, x_{1,2}, x_{1,3}, \ldots, x_{1,N}\}$
  $\{x_{2,1}, x_{2,2}, x_{2,3}, \ldots, x_{2,N}\}$
  …
  $\{x_{K,1}, x_{K,2}, x_{K,3}, \ldots, x_{K,N}\}$

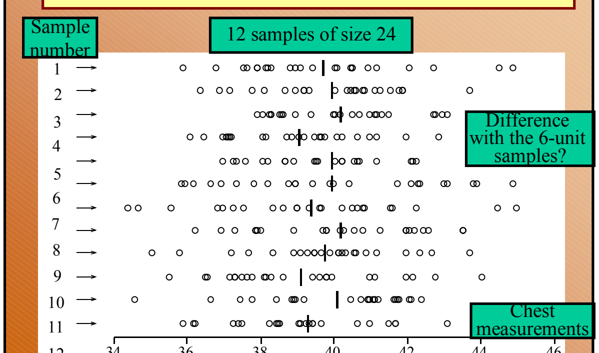
  K samples of size N. Data comes from a distr'n with μ, σ, but we're interested in mean/std-dev of sample average
- As the number of samples and the number of observations within each sample increase we get a better estimate of the true population parameter (say the mean). Scottish soldiers chest measurements example …

The sample mean has a sampling distribution

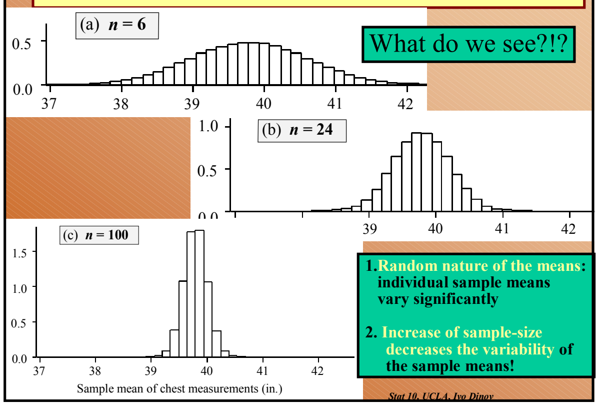
Sampling batches of 6 Scottish soldiers and taking chest measurements. Population $\mu = 39.8$ *in*, and $\sigma = 2.05$ *in*.



Twelve samples of size 24



Histograms from 100,000 samples, n=6, 24, 100



What do we see?!?

1. Random nature of the means: individual sample means vary significantly
2. Increase of sample-size decreases the variability of the sample means!

Mean and SD of the sampling distribution

E(*sample mean*) = Population *mean*

$$\text{SD}(\textit{sample mean}) = \frac{\text{Population } SD}{\sqrt{\text{Sample size}}}$$

$$\text{E}(\overline{X}) = \text{E}(X) = \mu, \quad \text{SD}(\overline{X}) = \frac{\text{SD}(X)}{\sqrt{n}} = \frac{\sigma}{\sqrt{n}}$$

Review

- We use both $\bar{x}$ and $\overline{X}$ to refer to a sample mean. For what purposes do we use the *former* and for what purposes do we use the *latter*? (sample mean estimate vs. estimator)
- What is meant by "the sampling distribution of $\overline{X}$"?

  (sampling variation – the observed variability in the process of taking random samples; sampling distribution – the real probability distribution of the random sampling process)
- How is the population mean of the sample average $\overline{X}$ related to the population mean of individual observations? (E($\overline{X}$) = Population mean)

Bias and Precision

- The bias in an estimator is the distance between between the *center* of the sampling distribution of the estimator and the true value of the parameter being estimated. In math terms, bias = $E(\hat{\Theta}) - \theta$, where theta, $\hat{\Theta}$, is the estimator, as a RV, of the true (unknown) parameter θ .
- Example, Why is the sample mean an **unbiased** estimate for the population mean? How about ¾ of the sample mean?

$$E(\hat{\Theta}) - \mu = E\left(\frac{1}{n}\sum_{k=1}^{n} X_k\right) - \mu = 0$$

$$E(\hat{\Theta}) - \mu = E\left(\frac{3}{4}\frac{1}{n}\sum_{k=1}^{n} X_k\right) - \mu = \frac{3}{4}\mu - \mu = \frac{\mu}{4} \neq 0, \quad \text{in general.}$$

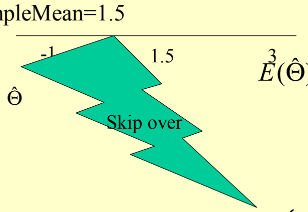
Bias and Precision

- X={1, -1, 3}, sample_mean =1, Estimator1=3/2 xSampleMean=1.5

-1 1.5 3

$\hat{\Theta}$

$E(\hat{\Theta}) - \theta$

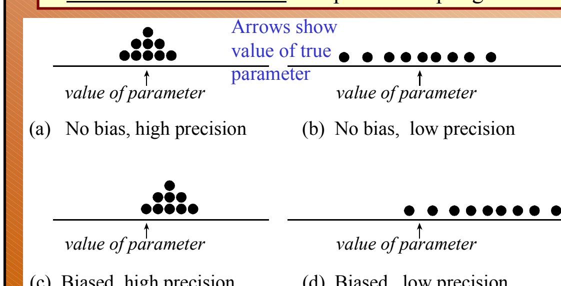


$$E(\hat{\Theta}) - \mu = E\left(\frac{3}{4}\frac{1}{n}\sum_{k=1}^{n} X_k\right) - \mu = \frac{3}{4}\mu - \mu = \frac{\mu}{4} \neq 0, \quad \text{in general.}$$

$$E(\hat{\Theta}) - \mu = E\left(\frac{1}{n}\sum_{k=1}^{n} X_k\right) - \mu = 0$$

Bias and Precision

- The precision of an estimator is a measure of how variable is the estimator in repeated sampling.



Standard error of an estimate

The ***standard error*** of any estimate $\hat{\theta}$ [denoted se($\hat{\theta}$)]

- estimates the variability of $\hat{\theta}$ values in repeated sampling and
- is a measure of the ***precision*** of $\hat{\theta}$. Example: $\overline{X}$, as an estimator of the population mean, μ.

$$SE(\overline{X}) = \frac{\sigma}{\sqrt{n}}, \text{ where } \overline{X} = \frac{1}{n}\sum_{k=1}^{n} X_k, \text{ and}$$

σ is the standard deviation of $\{X_k\}$, $1 \le k \le n$.

Review

- What is meant by the terms parameter and estimate.
- Is an estimator a RV?
- What is statistical inference? (process of making conclusions or making useful statements about unknown distribution parameters based on observed data.)
- What are bias and precision?
- What is meant when an estimate of an unknown parameter is described as unbiased?