UCLA STAT 251

Statistical Methods for the Life and Health Sciences

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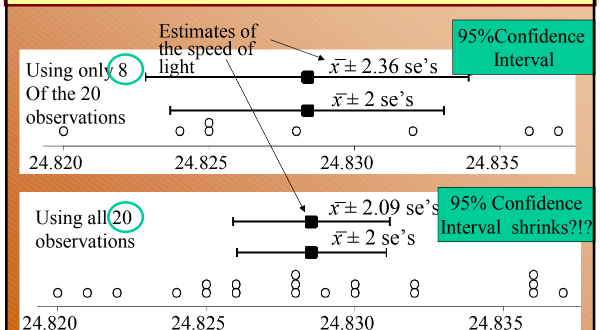
Confidence Intervals

20 replicated measurements to estimate the speed of light. Obtained by Simon Newcomb in 1882, by using distant (3.721 km) rotating mirrors.



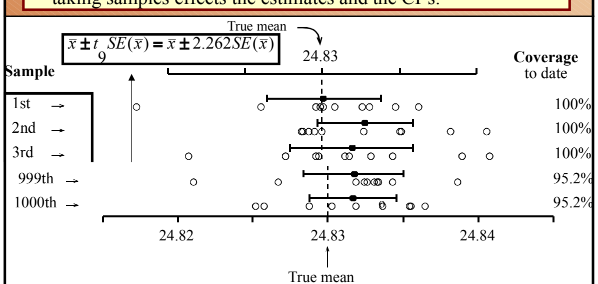
Passage time (10^{-6} seconds)

A 95% confidence interval

- A type of interval that contains the <u>true value of a parameter</u> for 95% of samples taken is called a ***95% confidence interval*** for that parameter, the ends of the CI are called *confidence limits*.
- (For the situations we deal with) a confidence interval (CI) for the true value of a parameter is given by

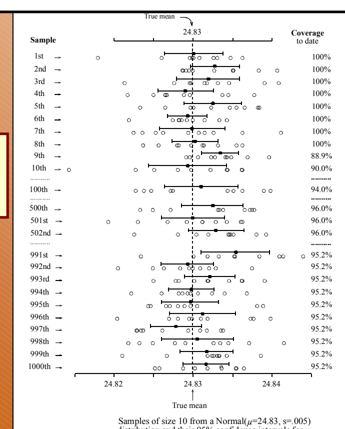
estimate $\pm$ ***t standard errors***

- CI are constructed using the sample $\bar{x}$ and s=SE. But diff. samples yield diff. estimates and ➔ diff. CI's?!?
- Below is a <u>computer simulation</u> showing how the process of taking samples effects the estimates and the CI's.



Samples of size 10 from a Normal(μ=24.83, s=.005) distribution and their 95% confidence intervals for μ..

Most of the table



How many of the previous samples contained the true mean?

Summary - CI for population mean

Confidence Interval for the true (population) mean μ:
sample mean $\pm$ ***t standard errors***

or $\bar{x} \pm t\,\text{se}(\bar{x})$, where $\text{se}(\bar{x}) = \frac{s_x}{\sqrt{n}}$ and $df = n-1$

Value of the Multiplier, t, for a 95% CI

df:	7	8	9	10	11	12	13	14	15	16	17
t:	2.365	2.306	2.262	2.228	2.201	2.179	2.160	2.145	2.131	2.120	2.110
df:	18	19	20	25	30	35	40	45	50	60	∞
t:	2.101	2.093	2.086	2.060	2.042	2.030	2.021	2.014	2.009	2.000	1.960

Effect of increasing the confidence level

99% CI, $\bar{x} \pm 2.576\,\text{se}(\bar{x})$

95% CI, $\bar{x} \pm 1.960\,\text{se}(\bar{x})$

90% CI, $\bar{x} \pm 1.645\,\text{se}(\bar{x})$

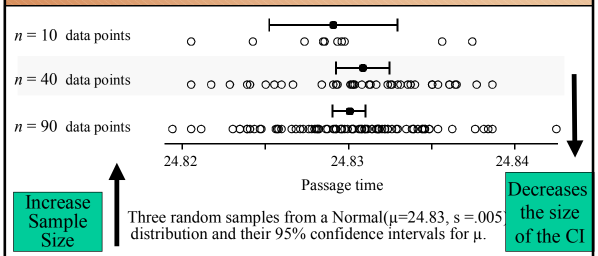
80% CI, $\bar{x} \pm 1.282\,\text{se}(\bar{x})$

Confidence Level Increase ↑ — Increases the size of the CI ↑

Why?

The greater the confidence level, the wider the interval

Effect of **increasing** the sample size



Three random samples from a Normal(μ=24.83, s =.005) distribution and their 95% confidence intervals for μ.

To ***double the precision*** we need ***four times*** as many observations.

Why ↑ in sample-size ↓ CI?

Confidence Interval for the true (population) mean μ:
sample mean $\pm$ ***t standard errors***

or $\bar{x} \pm t\,\text{se}(\bar{x})$, where $\text{se}(\bar{x}) = \frac{s_x}{\sqrt{n}}$ and $df = n-1$

CI for a population proportion

Confidence Interval for the true (population) proportion p:
sample proportion $\pm$ ***z standard errors***

or $\hat{p} \pm z\,\text{se}(\hat{p})$, where $\text{se}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

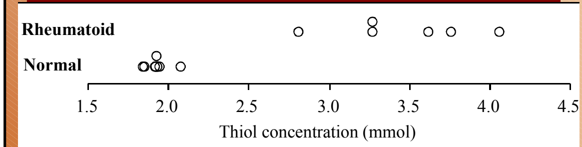
Example – higher blood thiol concentrations associated with rheumatoid arthritis?!?

Research question: Is the change in the Thiol status in the lysate of packed blood cells substantial to be indicative of a non trivial relationship between Thiol-levels and rheumatoid arthritis?

Thiol Concentration (mmol)

	Normal	Rheumatoid
	1.84	2.81
	1.92	4.06
	1.94	3.62
	1.92	3.27
	1.85	3.27
	1.91	3.76
	2.07	
Sample size	7	6
Sample mean	1.92143	3.46500
Sample standard deviation	0.07559	0.44049

Example – higher blood thiol concentrations with rheumatoid arthritis



Dot plot of Thiol concentration data.

Two groups of subjects are studied: 1. NC (normal controls)
2. RA (rheumatoid arthritis).

Observations: 1. The avg. levels of thiol seem diff. in NC & RA
2. NC and RA groups are separated completely.

Question: Is there statistical evidence that thiol-level correlates with the disease?

Difference between means

Confidence Interval for a difference between population means $(\mu_1 - \mu_2)$:

Difference between sample means
± t standard errors of the difference

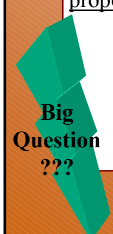
or $$\bar{x}_1 - \bar{x}_2 \pm t\ \mathrm{se}(\bar{x}_1 - \bar{x}_2)$$

Difference between proportions

Confidence Interval for a difference between population proportions $(p_1 - p_2)$:

Difference between sample proportions
± z standard errors of the difference

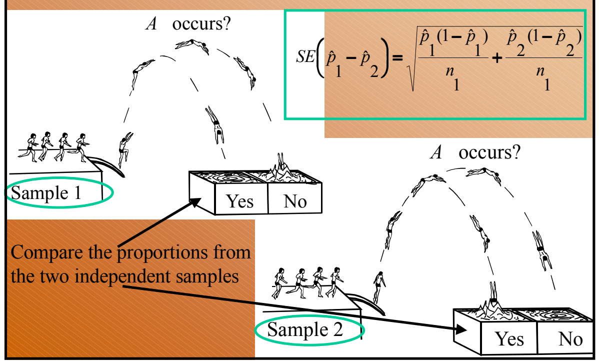
$$\hat{p}_1 - \hat{p}_2 \pm z\ \mathrm{se}(\hat{p}_1 - \hat{p}_2)$$



How do we compute the SE($\hat{p}_1 - \hat{p}_2$) for different cases?

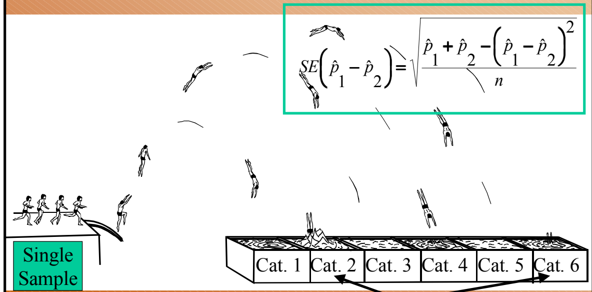
1. Proportions from 2 independent samples

$$SE\left(\hat{p}_1 - \hat{p}_2\right) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_1}}$$



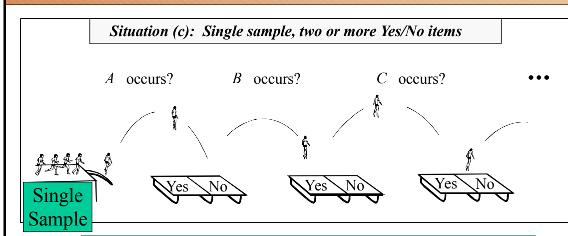
2. Single sample, several response categories

$$SE\left(\hat{p}_1 - \hat{p}_2\right) = \sqrt{\frac{\hat{p}_1 + \hat{p}_2 - \left(\hat{p}_1 - \hat{p}_2\right)^2}{n}}$$



Compare different proportions from the same sample

3. Single sample, several yes/no items



From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 2000.

$$SE\left(\hat{p}_1 - \hat{p}_2\right) = \sqrt{\frac{\mathrm{Min}\left(\hat{p}_1 + \hat{p}_2 ; 2 - \hat{p}_1 - \hat{p}_2\right) - \left(\hat{p}_1 - \hat{p}_2\right)^2}{n}}$$

SE's for the 3 cases of differences in proportion

(a) **Proportions from two independent samples** of sizes n_1 and n_2, respectively

$$se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

(b) **One sample of size n, several response categories**

$$se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1 + \hat{p}_2 - (\hat{p}_1 - \hat{p}_2)^2}{n}}$$

(c) **One sample of size n, many Yes/No items**

$$se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{Min(\hat{p}_1 + \hat{p}_2, \hat{q}_1 + \hat{q}_2) - (\hat{p}_1 - \hat{p}_2)^2}{n}}$$

where $\hat{q}_1 = 1 - \hat{p}_1$ and $\hat{q}_2 = 1 - \hat{p}_2$

1998 Back-to-school survey – smoking/drinking 2000 students participating in the survey

Characteristics by Smoking and Drinking Status

(Table entry is % of group saying yes)	Smoker	Nonsmoker	Drinker	Nondrinker
Get mostly A's or B's?	41	68		
Read 1 or more hours/day?	54	72	56	75
Get drunk at least once a month?	63	10		
Have smoked Marijuana?	79	14	52	12
Likely to try illegal drug in future?	42	14	35	11
n	130	870	260	740

1 Sample many yes/no Answers -
compare proportions of smokers who get a drink at least once a month(63) With proportion of smokers who have smoked marijuana (79). Smokers group, say, are not forced to choose 1 category out of a set of categories as in the previous situation.

Sample size - proportion

- **For a 95% CI,** $\text{margin} = 1.96 \times \sqrt{\hat{p}(1-\hat{p})/n}$
- **Sample size for a desired margin of error:** For a margin of error no greater than m, use a sample size of approximately

$$n = \left(\frac{z}{m}\right)^2 \times p^*(1-p^*)$$

- p^* is a guess at the value of the proportion -- err on the side of being too close to 0.5
- z is the multiplier appropriate for the confidence level
- m is expressed as a proportion (between 0 and 1), not a percentage (basically, What's n, so that m >= margin?)

Sample size -- mean

- **Sample size for a desired margin of error:** For a margin of error no greater than m, use a sample size of approximately

$$n = \left(\frac{z\sigma^*}{m}\right)^2$$

- σ^* is an estimate of the variability of individual observations
- z is the multiplier appropriate for the confidence level

Covariance – a measure of LINEAR association between two variables, X & Y

$$E(aY_1 + bY_2) = aE(Y_1) + bE(Y_2) = a\mu_1 + b\mu_2$$

$$Var(aY_1 + bY_2 + c) = a^2Var(Y_1) + b^2Var(Y_2) + 2abCov(Y_1; Y_2)$$

$$Cov(Y_1; Y_2) = E[(Y_1 - \mu_1)(Y_2 - \mu_2)]$$

HypothesisTesting4a.ppt and then:

F_Chi2_dist_Ch4_6.pdf Variance estimates/CI's