Stat 251 /OBEE 216

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Project 1 Solution

1.1

X = the total number of T-bases observed in the compiste experiment
X is a discrete random variable which its values will take on {0, 1, 2, 3}

Ignoring the stopping criteria, the entire sample space will be $4^4 = 256$ total possible permutations. It would be rather time consuming and tedious to list all 256 permutations. So we will create a new R.V., Y.

Y = the total number of bases observed until a stopping criterion is met.
Y is a discrete R.V. which its values will take on {2, 3, 4}

Conditioning X on Y, now we can calculate their joint probabilities.

Y = 2

$X = 0$ $\qquad T'G$

$$P(X=0|Y=2) = \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{4^2}$$

$X = 1$ $\qquad TG$

$$P(X=1|Y=2) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{4^2}$$

Y = 3

$X = 0$ $\qquad T' \begin{matrix} T' \\ G' \end{matrix} G$

$$P(X=0|Y=3) = \frac{3}{4} \cdot \frac{2}{4} \cdot \frac{1}{4} = \frac{6}{4^3}$$

$X = 1$ $\qquad T \begin{matrix} T' \\ G' \end{matrix} G \qquad TAA \qquad T'TG$

$$P(X=1|Y=3) = \left(\frac{1}{4} \cdot \frac{2}{4} \cdot \frac{1}{4}\right) + \left(\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}\right) + \left(\frac{3}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}\right) = \frac{6}{4^3}$$

$X = 2$ $\qquad TTG$

$$P(X=2|Y=3) = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{4^3}$$

$X = 3$ $\qquad TTT$

$$P(X=3|Y=3) = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{4^3}$$

Y = 4

X = 0

$$T^{'}\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}T^{'}$$

$$P(X=0|Y=4)=\frac{3}{4}\cdot\frac{2}{4}\cdot\frac{2}{4}\cdot\frac{3}{4}=\frac{36}{4^4}$$

X = 1

$$T\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}T^{'}\ \square\ TAA\bullet$$

$$P(X=1|Y=4)=\left(\frac{1}{4}\cdot\frac{2}{4}\cdot\frac{2}{4}\cdot\frac{3}{4}\right)-\left(\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{4}{4}\right)$$

$$T^{'}T\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}T^{'}+T^{'}\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}TT^{'}+T^{'}\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}T$$

$$+\left(\frac{3}{4}*\frac{1}{4}*\frac{2}{4}*\frac{3}{4}\right)+\left(\frac{3}{4}*\frac{2}{4}*\frac{1}{4}*\frac{3}{4}\right)+\left(\frac{3}{4}*\frac{2}{4}*\frac{2}{4}*\frac{1}{4}\right)=\frac{56}{4^4}$$

X = 2

$$TT\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}T^{'}+T\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}TT^{'}+T\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}T$$

$$P(X=2|Y=4)=\left(\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{2}{4}\cdot\frac{3}{4}\right)+\left(\frac{1}{4}\cdot\frac{2}{4}\cdot\frac{1}{4}\cdot\frac{3}{4}\right)+\left(\frac{1}{4}\cdot\frac{2}{4}\cdot\frac{2}{4}\cdot\frac{1}{4}\right)$$

$$T^{'}TTT^{'}+T^{'}T\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}T+T^{'}\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}TT$$

$$+\left(\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{4}\right)+\left(\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{2}{4}\cdot\frac{1}{4}\right)+\left(\frac{3}{4}\cdot\frac{2}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\right)=\frac{37}{4^4}$$

X = 3

$$T^{'}TTT+T\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}TT+TT\genfrac{}{}{0pt}{}{T^{'}}{G^{'}}T$$

$$P(X=3|Y=4)=\left(\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\right)+\left(\frac{1}{4}\cdot\frac{2}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\right)+\left(\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{2}{4}\cdot\frac{1}{4}\right)=\frac{7}{4^4}$$

Using the conditional probabilities, the probability distribution of X is a probability mass function. The marginal probability of each X is as followed:

$$P(X=0)=P(X=0|Y=2)+P(X=0|Y=3)+P(X=0|Y=4)=\frac{48}{4^4}+\frac{24}{4^4}+\frac{36}{4^4}=\frac{108}{4^4}$$

$$P(X=1)=P(X=1|Y=2)+P(X=1|Y=3)+P(X=1|Y=4)=\frac{16}{4^4}+\frac{24}{4^4}+\frac{56}{4^4}=\frac{96}{4^4}$$

$$P(X=2)=P(X=2|Y=3)+P(X=2|Y=4)=\frac{4}{4^4}+\frac{37}{4^4}=\frac{41}{4^4}$$

$$P(X=3)=P(X=3|Y=3)+P(X=3|Y=4)=\frac{4}{4^4}+\frac{7}{4^4}=\frac{11}{4^4}$$

Mean:

$$\mu = \sum x_i P(X = x_i) = 0 * \frac{108}{4^4} + 1 * \frac{96}{4^4} + 2 * \frac{41}{4^4} + 3 * \frac{11}{4^4} = 0.8242$$

Standard Deviation:

$$\sigma = \sqrt{\sum (x_i - \mu)^2 P(X = x_i)}$$

$$= \sqrt{(0 - 0.8242)^2 \frac{108}{4^4} + (1 - 0.8242)^2 \frac{96}{4^4} + (2 - 0.8242)^2 \frac{41}{4^4} + (3 - 0.8242)^2 \frac{11}{4^4}} = 0.8503$$

1.2

With the given information, the expected number of acquaintances met per hour for an 8 hours day on campus follows a Poisson Process with parameter $\lambda = \frac{25}{8}$.

X = the time that passes until the **first** acquaintance is met

X will follow an exponential distribution with parameter $\lambda = \frac{25}{8}$.

Thus,

$$P(1 < X < 3) = \int_1^3 \lambda e^{-\lambda x} = \left(1 - e^{-\frac{25}{8} * 3}\right) - \left(1 - e^{-\frac{25}{8} * 1}\right) = 0.9999 - 0.9561 = 0.0438$$

Mean for exponential distribution is

$$\mu = \frac{1}{\lambda} = \frac{8}{25}$$