APPENDIX A

Common Distributions

Discrete Distributions

Binomial

$$
\begin{aligned}
p(k) &= \binom{n}{k} p^k (1-p)^{n-k}, \qquad k = 0, 1, \ldots, n \\
E(X) &= np \\
\mathrm{Var}(X) &= np(1-p) \\
M(t) &= (1 - p + pe^t)^n
\end{aligned}
$$

Geometric

$$
\begin{aligned}
p(k) &= p(1-p)^{k-1}, \qquad k = 1, \ldots \\
E(X) &= \frac{1}{p} \\
\mathrm{Var}(X) &= \frac{1-p}{p^2} \\
M(t) &= \frac{e^t p}{1-(1-p)e^t}
\end{aligned}
$$

Negative Binomial

$$
\begin{aligned}
p(k) &= \binom{k-1}{r-1} p^r (1-p)^{k-r}, \qquad k = r, r+1, \ldots \\
E(X) &= \frac{r}{p} \\
\mathrm{Var}(X) &= \frac{r(1-p)}{p^2} \\
M(t) &= \left(\frac{e^t p}{1-(1-p)e^t} \right)^r
\end{aligned}
$$

Poisson

$$p(k) = \frac{\lambda^k e^{-\lambda}}{k!}, \qquad k = 0, 1, \ldots$$
$$E(X) = \lambda$$
$$\mathrm{Var}(X) = \lambda$$
$$M(t) = e^{\lambda(e^t - 1)}$$

Continuous Distributions

Normal

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \qquad -\infty < x < \infty$$
$$E(X) = \mu$$
$$\mathrm{Var}(X) = \sigma^2$$
$$M(t) = e^{\mu t} e^{\sigma^2 t^2/2}$$

Gamma

$$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, \qquad x \geq 0$$
$$E(X) = \frac{\alpha}{\lambda}$$
$$\mathrm{Var}(X) = \frac{\alpha}{\lambda^2}$$
$$M(t) = \left(\frac{\lambda}{\lambda - t}\right)^\alpha, \qquad t < \lambda$$

Exponential (Special Case of Gamma with $\alpha = 1$)

Chi-Square with n Degrees of Freedom (Special Case of Gamma with $\alpha = n/2$, $\lambda = \frac{1}{2}$)

Uniform

$$f(x) = 1, \qquad 0 \leq x \leq 1$$
$$E(X) = \tfrac{1}{2}$$
$$\mathrm{Var}(X) = \tfrac{1}{12}$$
$$M(t) = \frac{e^t - 1}{t}$$

Beta

$$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}, \qquad 0 \le x \le 1$$

$$E(X) = \frac{a}{a+b}$$

$$\text{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

$M(t)$ is not useful.