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BETAS AND THEIR REGRESSION TENDENCIES

MARSHALL E. BLUME*

I. INTRODUCTION

A PREVIOUS STUDY [3] showed that estimated beta coefficients, at least in the context of a portfolio of a large number of securities, were relatively stationary over time. Nonetheless, there was a consistent tendency for a portfolio with either an extremely low or high estimated beta in one period to have a less extreme beta as estimated in the next period. In other words, estimated betas exhibited in that article a tendency to regress towards the grand mean of all betas, namely one. This study will examine in further detail this regression tendency.[1]

The next section presents evidence showing the existence of this regression tendency and reviews the conventional reasons given in explanation [1], [4], [5]. The following section develops a formal model of this regression tendency and finds that the conventional analysis of this tendency is, if not incorrect, certainly misleading. Accompanying this theoretical analysis are some new empirical results which show that a major reason for the observed regression is real non-stationarities in the underlying values of beta and that the so-called "order bias" is not of dominant importance.

II. THE CONVENTIONAL WISDOM

If an investor were to use estimated betas to group securities into portfolios spanning a wide range of risk, he would more than likely find that the betas estimated for the very same portfolios in a subsequent period would be less extreme or closer to the market beta of one than his prior estimates. To illustrate, assume that the investor on July 1, 1933, had at his disposal an estimate of beta for each common stock which had been listed on the NYSE (New York Stock Exchange) for the prior seven years, July 1926-June 1933. Assume further that each estimate was derived by regressing the eighty-four monthly relatives covering this seven-year period upon the corresponding values for the market portfolio.[2]

If this investor, say, desired equally weighted portfolios of 100 securities, he might group those 100 securities with the smallest estimates of beta together to form a portfolio. Such a portfolio would of all equally

* Professor of Finance, University of Pennsylvania. The author wishes to thank Professors John Bildersee and Harry Markowitz for their helpful comments and the Rodney L. White Center for financial support.

1. Quite apart from this regression tendency, it is reasonable to suppose that betas do change over time in systematic ways in response to certain changes in the structure of companies.

2. Such regressions were calculated only for securities with complete data. The relative for the market portfolio was measured by Fisher's Combination Link Relative [6].

weighted portfolios have the smallest possible estimated portfolio beta since an estimate of such a portfolio beta can be shown to be an average of the estimates for the individual securities [2, p. 169]. To cover a wide range of portfolio betas, this investor might then form a second portfolio consisting of the 100 securities with the next smallest estimates of beta, and so on.

Using the securities available as of June 1933, this investor could thus obtain four portfolios of 100 securities apiece with no security in common. Estimated over the same seven-year period, July 1926-June 1933, the betas for these portfolios[3] would have ranged from 0.50 to 1.53. Similar portfolios can be constructed for each of the next seven-year periods through 1954 and their portfolio betas calculated. Table 1 contains these estimates under the heading "Grouping Period."

The betas for these same portfolios, but reestimated using the monthly portfolio relatives adjusted for delistings from the seven years following the grouping period, illustrate the magnitude of the regression tendency.[4] Whereas the portfolio betas as estimated, for instance, in the grouping period 1926-33 ranged from 0.50 to 1.53, the betas as estimated for these same portfolios in the subsequent seven-year period 1933-40 ranged only from 0.61 to 1.42. The results for the other periods display a similar regression tendency.

An obvious explanation of this regression tendency is that for some unstated economic or behavioral reasons, the underlying betas do tend to regress towards the mean over time.[5] Yet, even if the true betas were constant over time, it has been argued that the portfolio betas as estimated in the grouping period would as a statistical artifact tend to be more extreme than those estimated in a subsequent period. This bias has sometimes been termed an order or selection bias.

The frequently given intuitive explanation of this order bias [1], [4], [5], parallels the following: Consider the portfolio formed of the 100 securities with the lowest estimates of beta. The estimated portfolio beta might be expected to understate the true beta or equivalently be expected to be measured with negative error. The reason the measurement error might

3. These portfolio betas were derived by averaging the 100 estimates for the individual securities. Alternatively, as [2] shows, the same number would be obtained by regressing the monthly portfolio relatives upon the market index where the portfolio relatives are calculated assuming an equal amount invested in each security at the beginning of each month.

4. These portfolio betas were calculated by regressing portfolio relatives upon the market relatives. The portfolio relatives were taken to be the average of the monthly relatives of the individual securities for which relatives were available. These relatives represent those which would have been realized from an equally-weighted, monthly rebalancing strategy in which a delisted security is sold at the last available price and the proceeds reinvested equally in the remaining securities. This rather complicated procedure takes into account delisted securities and therefore avoids any survivorship bias. In [3], the securities analyzed were required to be listed on the NYSE throughout both the grouping period and the subsequent period, so that there was a potential survivorship bias. Nonetheless, the results reported there are in substantive agreement with the results in Table 1.

5. If the betas are continually changing over time, an estimate of beta as provided by a simple regression must be interpreted with considerable caution. For example, if the true beta followed a linear time trend, it is easily shown that the estimated beta can be interpreted as an unbiased estimate of the beta in the middle of the sample period. A similar interpretation would not in general hold if, for instance, the true beta followed a quadratic time trend.

TABLE 1
BETA COEFFICIENTS FOR PORTFOLIOS OF 100 SECURITIES

Portfolio	Grouping Period	First Subsequent Period
	7/26-6/33	7/33-6/40
1	0.50	0.61
2	0.85	0.96
3	1.15	1.24
4	1.53	1.42
	7/33-6/40	7/40-6/47
1	0.38	0.56
2	0.69	0.77
3	0.90	0.91
4	1.13	1.12
5	1.35	1.31
6	1.68	1.69
	7/40-6/47	7/47-6/54
1	0.43	0.60
2	0.61	0.76
3	0.73	0.88
4	0.86	0.99
5	1.00	1.10
6	1.21	1.21
7	1.61	1.36
	7/47-6/54	7/54-6/61
1	0.36	0.57
2	0.61	0.71
3	0.78	0.88
4	0.91	0.96
5	1.01	1.03
6	1.13	1.13
7	1.26	1.24
8	1.47	1.32
	7/54-6/61	7/61-6/68
1	0.37	0.62
2	0.56	0.68
3	0.72	0.85
4	0.86	0.85
5	0.99	0.95
6	1.11	0.98
7	1.23	1.07
8	1.43	1.25

be expected to be negative may best be explored by analyzing how a security might happen to have one of the 100 lowest estimates of beta. First, if the true beta were in the lowest hundred, the estimated beta would fall in the lowest 100 estimates only if the error in measuring the beta were not too large which roughly translates into more negative than positive errors. Second, if the true beta were not in the lowest 100, the estimated beta might still be in the lowest 100 estimates if it were measured with a sufficiently large negative error.[6]

Thus, the negative errors in the 100 smallest estimates of beta might be expected to outweigh the positive errors. The same argument except in reverse would apply to the 100 largest estimates. Indeed, it would seem that any portfolio of securities stratified by estimates of beta for which the average of these estimates is not the grand mean of all betas, namely 1.0, would be subject to some order bias. It would also seem that the absolute magnitude of this order bias should be greater, the further the average estimate is from the grand mean. The next section formalizes this intuitive argument and suggests that, if it is not incorrect, it is certainly misleading as to the source of the bias.

III. A Formal Model

The intuitive explanation of the order bias just given would seem to suggest that the way in which the portfolios are formed caused the bias. This section will argue that the bias is present in the estimated betas for the individual securities and is not induced by the way in which the portfolios are selected. Following this argument will be an analysis of the extent to which this order bias accounts for the observed regression tendency in portfolio betas over time.

A numerical example will serve to illustrate the logic of the subsequent argument and to introduce some required notation.[7] Assume for the moment that the possible values of beta for an individual security i in period t, β_{it}, are 0.8, 1.0 and 1.2 and that each of these values is equally likely. Assume further that in estimating a beta for an individual security, there is a 0.6 probability that the estimate $\hat{\beta}_{it}$ contains no measurement error, a 0.2 probability that it understates the true β_{it} by 0.2, and a 0.2 probability that it overstates the true value by 0.2. Now in a sample of ten securities whose true betas were all say 0.8, one would expect two estimates of beta to be 0.6, six to be 0.8, and two to be 1.0. These numbers have been transcribed to the first row of Table 2. The second and third rows are similarly constructed by first assuming that the ten securities all had a true value of 1.0 and then of 1.2.

The rows of Table 2 thus correspond to the distribution of the estimated beta, $\hat{\beta}_{it}$, conditional on the true value, β_{it}. It might be noted that the expectation of $\hat{\beta}_{it}$ conditional on β_{it}, $E(\hat{\beta}_{it} \mid \beta_{it})$, is β_{it}. However, in a sampling situation, an investigator would be faced with an estimate of beta and would want to assess the distribution of the true β_{it} conditional on the estimated $\hat{\beta}_{it}$. Such conditional distributions correspond to the columns of Table 2. It is easily verified that the expectation of β_{it} conditional on $\hat{\beta}_{it}$, $E(\beta_{it} \mid \hat{\beta}_{it})$ is generally not $\hat{\beta}_{it}$. For example, if $\hat{\beta}_{it}$ were

6. It is theoretically possible that the estimated beta for a security whose true beta does not fall into the lowest 100 to be in the lowest 100 estimates with a positive measurement error if the betas for some of the improperly classified securities are measured with sufficiently large positive errors.

7. The author is indebted to Harry Markowitz for suggesting this numerical example as a way of clarifying the subsequent formal development.

TABLE 2
NUMBER OF SECURITIES CROSS CLASSIFIED BY β_{it} AND $\hat{\beta}_{it}$

		$\hat{\beta}_{it}$				
		.6	.8	1.0	1.2	1.4
	.8	2	6	2		
β_{it}	1.0		2	6	2	
	1.2			2	6	2

0.8, $E(\beta_{it} \mid \hat{\beta}_{it} = 0.8)$ would be 0.85 since with this estimate the true beta would be 0.8 with probability 0.75 or 1.0 with probability 0.25.[8]

The estimate $\hat{\beta}_{it}$, therefore, would typically be biased, and it is biased whether or not portfolios are formed. The effect of forming large portfolios is to reduce the random component in the estimate, so that the difference between the estimated portfolio beta and the true portfolio beta can be ascribed almost completely to the magnitude of the bias.

In the spirit of this example, the paper will now develop explicit formulae for the order bias and real non-stationarities over time. Let it be assumed that the betas for individual securities in period t, β_{it}, can be thought of as drawings from a normal distribution with a mean of 1.0 and variance $\sigma^2(\beta_{it})$. The corresponding assumption for the numerical example just discussed would be a trinomial distribution with equal probabilities for each possible value of β_{it}.

Let it additionally be assumed that the estimate, $\hat{\beta}_{it}$, measures β_{it} with error η_{it}, a mean-zero independent normal variate, so that $\hat{\beta}_{it}$ is given by the sum of β_{it} and η_{it}. It immediately follows that β_{it} and $\hat{\beta}_{it}$ are distributed by a bivariate normal distribution. It might be noted that, as formulated, $\sigma^2(\eta_{it})$ need not equal $\sigma^2(\eta_{jt})$, $i \neq j$. Since the empirical work will assume equality, the subsequent theoretical work will also make this assumption even though for the most part it is not necessary. The final assumption is that β_{it} and β_{it+1} are distributed as bivariate normal variates. Because η_{it} is independently distributed, $\hat{\beta}_{it}$ and β_{it+1} will be distributed by a bivariate normal distribution.

That $\hat{\beta}_{it}$ and β_{it+1} are bivariate normal random variables, each with a mean of 1.0, implies the following regression

$$E(\beta_{it+1} \mid \hat{\beta}_{it}) - 1 = \frac{\mathrm{Cov}\,(\beta_{it+1}, \hat{\beta}_{it})}{\sigma^2(\hat{\beta}_{it})}\,(\hat{\beta}_{it} - 1). \tag{1}$$

This regression is similar to the procedure proposed in Blume [3] to adjust the estimated betas for the regression tendency. That procedure was to regress estimates of beta for individual securities from a later period on estimates from an earlier period and to use the coefficients from this regression to adjust future estimates.[9] The empirical evidence

8. For further and more detailed discussion of the distinction between $E(\beta_{it} \mid \hat{\beta}_{it})$ and $E(\hat{\beta}_{it} \mid \beta_{it})$, the reader is referred to Vasicek [7].

9. That the regression of estimated betas from a later period on estimates from an earlier period is similar to (1) follows from noting that $E(\hat{\beta}_{it+1} \mid \hat{\beta}_{it})$ equals $E(\beta_{it+1} \mid \hat{\beta}_{it})$ and that $\mathrm{Cov}(\hat{\beta}_{it+1}, \hat{\beta}_{it})$ equals $\mathrm{Cov}(\beta_{it+1}, \hat{\beta}_{it})$. In [3], the grand mean of all betas was estimated in each period and was not assumed equal to 1.0.

presented there indicated that this procedure did improve the accuracy of estimates of future betas, though no claim was made that there might not be better ways to adjust for the regression tendency.

The coefficient of $(\hat{\beta}_{it} - 1)$ in (1) can be broken down into two components: one of which would correspond to the so-called order bias and the other to a true regression tendency. To achieve this result, note that the covariance of β_{it+1} and $\hat{\beta}_{it}$ is given by $\text{Cov}(\beta_{it+1}, \beta_{it} + \eta_{it})$, which because of the assumed independence of the errors, reduces to the covariance of β_{it+1} and β_{it}. Making this substitution and replacing $\text{Cov}(\beta_{it+1}, \beta_{it})$ by $\rho(\beta_{it+1}, \beta_{it})\sigma(\beta_{it+1})\sigma(\beta_{it})$, (1) becomes

$$E(\beta_{it+1} \mid \hat{\beta}_{it}) - 1 = \frac{\rho(\beta_{it+1}, \beta_{it})\sigma(\beta_{it+1})\sigma(\beta_{it})}{\sigma^2(\hat{\beta}_{it})} (\hat{\beta}_{it} - 1). \tag{2}$$

The ratio of $\sigma(\beta_{it})\sigma(\beta_{it+1})$ to $\sigma^2(\hat{\beta}_{it})$ might be identified with the order bias and the correlation of β_{it} and β_{it+1} with a true regression.

If the underlying values of beta are stationary over time, the correlation of successive values will be 1.0 and the standard deviations of β_{it} and β_{it+1} will be the same. Assuming such stationarity and noting then that β_{it+1} equals β_{it}, equation (2) can be rewritten as[10]

$$\begin{aligned} E(\beta_{it+1} \mid \hat{\beta}_{it}) - 1 &= E(\beta_{it} \mid \hat{\beta}_{it}) - 1 \\ &= \frac{\sigma^2(\beta_{it})}{\sigma^2(\hat{\beta}_{it})} (\hat{\beta}_{it} - 1). \end{aligned} \tag{3}$$

Since $\sigma^2(\beta_{it})$ would be less than $\sigma^2(\hat{\beta}_{it})$ if beta is measured with any error, the coefficient of $(\hat{\beta}_{it} - 1)$ would be less than 1.0. This means that the true beta for a security would be expected to be closer to one than the estimated value. In other words, an estimate of beta for an individual security except for an estimate of 1.0 is biased.[11]

10. Equation (3) can be derived alternatively from the assumption that β_{it} and $\hat{\beta}_{it}$ are bivariate normal variables and under the assumption of stationarity β_{it} will equal β_{it+1}. Vasicek [7] has developed using Bayes' Theorem, an expression for $E(\beta_{it}|\hat{\beta}_{it})$ which can be shown to be mathematically identical to the right hand side of (3): He observed that the procedure used by Merrill Lynch, Pierce, Fenner and Smith, Inc. in their Security Risk Evaluation Service is similar to his expression if $\sigma^2(\eta_{it})$ is assumed to be the same for all securities. Merrill Lynch's procedure, as he presented it, is to use the coefficient of the cross-sectional regression of $(\hat{\beta}_{it+1} - 1)$ on $(\hat{\beta}_{it} - 1)$ to adjust future estimates. This adjustment mechanism is in fact the same as (1) or (2) which shows that such a cross sectional regression takes into account real changes in the underlying betas. Only if betas were stationary over time would his formula be similar to Merrill Lynch's.

11. The formula for order bias given by (3) is similar to that which measures the bias in the estimated slope coefficient in a regression on one independent variable measured with error. Explicitly, consider the regression, $y = bx + \epsilon$, where ϵ is an independent mean-zero normal disturbance and both y and x are measured in deviate form. Now if x is measured with independent mean-zero error η and y is regressed on $x + \eta$, it is well known that the estimated coefficient, $\hat{b}$, will be biased toward zero and the probability limit of $\hat{b}$ is $\dfrac{b}{1 + \dfrac{\sigma^2(\eta)}{\sigma^2(x)}}$. This expression can be rewritten as $\dfrac{\sigma^2(x)}{\sigma^2(x + \eta)}$ b. Interpreting x as the true beta less 1.0, the correspondence to (3) is obvious. In this type of regression, one could either adjust the independent variables themselves for bias and thus obtain an unbiased estimate of the regression coefficient or run the regression on the unadjusted variables and then adjust the regression coefficient. The final coefficient will be the same in either case.

In light of this discussion, the paper now reexamines the empirical results of the previous section. The initial task will be to adjust the portfolio betas in the grouping periods for the order bias. After making this adjustment, it will be apparent that much of the regression tendency observed in Table 1 remains. Thus, if (2) is valid, the value of the correlation coefficient is probably not 1.0. The statistical properties of estimates of the portfolio betas in both the grouping and subsequent periods will be examined. The section ends with an additional test that gives further confirmation that much of the regression tendency stems from true non-stationarities in the underlying betas.

To adjust the estimates of beta in the grouping periods for the order bias using (3) would require estimates of the ratio of $\sigma^2(\beta_{it})$ to $\sigma^2(\hat{\beta}_{it})$. The sample variance calculated from the estimated betas for all securities in a particular cross-section provides an estimate of $\sigma^2(\hat{\beta}_{it})$. An estimate of $\sigma^2(\beta_{it})$ can be derived as the difference between estimates of $\sigma^2(\hat{\beta}_{it})$ and $\sigma^2(\eta_{it})$. If the variance of the error in measuring an individual beta is the same for every security, $\sigma^2(\eta_{it})$ can be estimated as the average over all securities of the squares of the standard error associated with each estimated beta.

In conformity with these procedures, estimates of the ratio of $\sigma^2(\beta_{it})$ to $\sigma^2(\hat{\beta}_{it})$ for the five seven-year periods from 1926 through 1961 were respectively 0.92, 0.92, 0.89, 0.82, and 0.75. In other words, an unbiased estimate of the underlying beta for an individual security should be some eight to twenty-five per cent closer to 1.0 than the original estimate. For instance, if $\sigma^2(\beta_{it})/\sigma^2(\hat{\beta}_{it})$ were 0.9 and if $\hat{\beta}_{it}$ were 1.3, an unbiased estimate would be 1.27.

To determine whether the order bias accounted for all of the regression, the estimated betas for the individual securities were adjusted for the order bias using (3) and the appropriate value of the ratio. For the same portfolios of 100 securities examined in the previous section, portfolio betas for the grouping period were recalculated as the average of these adjusted betas. It might be noted that these adjusted portfolio betas could alternatively be obtained by adjusting the unadjusted portfolio betas directly. These adjusted portfolio betas are given in Table 3. For the reader's convenience, the unadjusted portfolio betas and those estimated in the subsequent seven years are reproduced from Table 1.

Before comparing these estimates, let us for the moment consider the statistical properties of the portfolio betas, first in the grouping period and then in the subsequent period. Though unadjusted estimates of the portfolio betas in the grouping period may be biased, they would be expected to be highly "reliable" as that term is used in psychometrics. Thus, regardless of what these estimates measure, they measure it accurately or more precisely their values approximate those which would be expected conditional on the underlying population and how they are calculated. For equally-weighted portfolios, the larger the number of securities, the more reliable would be the estimate.

Specifically, for an equally-weighted portfolio of 100 securities, the standard deviation of the error in the portfolio beta would be one-tenth

TABLE 3

BETA COEFFICIENTS FOR PORTFOLIOS OF 100 SECURITIES

Portfolio	Grouping Period: Unadjusted for Order Bias	Grouping Period: Adjusted for Order Bias	First Subsequent Period	Second Subsequent Period
	7/26-6/33		7/33-6/40	7/40-6/47
1	0.50	.54	0.61	0.73
2	0.85	.86	0.96	0.92
3	1.15	1.14	1.24	1.21
4	1.53	1.49	1.42	1.47
	7/33-6/40		7/40-6/47	7/47-6/54
1	0.38	.43	0.56	0.53
2	0.69	.72	0.77	0.86
3	0.90	.91	0.91	0.96
4	1.13	1.12	1.12	1.11
5	1.35	1.32	1.31	1.29
6	1.68	1.63	1.69	1.40
	7/40-6/47		7/47-6/54	7/54-6/61
1	0.43	.50	0.60	0.73
2	0.61	.65	0.76	0.88
3	0.73	.76	0.88	0.93
4	0.86	.88	0.99	1.04
5	1.00	1.00	1.10	1.12
6	1.21	1.19	1.21	1.14
7	1.61	1.54	1.36	1.20
	7/47-6/54		7/54-6/61	7/61-6/68
1	0.36	.48	0.57	0.72
2	0.61	.68	0.71	0.79
3	0.78	.82	0.88	0.88
4	0.91	.93	0.96	0.92
5	1.01	1.01	1.03	1.04
6	1.13	1.10	1.13	1.02
7	1.26	1.21	1.24	1.08
8	1.47	1.39	1.32	1.15
	7/54-6/61		7/61-6/68	
1	0.37	.53	0.62	
2	0.56	.67	0.68	
3	0.72	.79	0.85	
4	0.86	.89	0.85	
5	0.99	.99	0.95	
6	1.11	1.08	0.98	
7	1.23	1.17	1.07	
8	1.43	1.32	1.25	

the standard error of the estimated betas for individual securities providing the errors in measuring these individual betas were independent of each other. During the 1926-33 period, the average standard error of betas for individual securities was 0.12 so that the standard error of the portfolio beta would be roughly 0.012. The average standard error for individual securities increased gradually to 0.20 in the period July 1954-June 1961. For the next seven-year period ending June 1968, the average declined to 0.17.

As pointed out, standard errors for portfolio betas calculated from those for individual securities assume independence of the errors in estimates. The standard error for a portfolio beta can however be calculated directly without making this assumption of independence by regressing the portfolio returns on the market index. The standard error for the portfolio of the 100 securities with the lowest estimates of beta in the July 1926-June 1933 period was for instance, 0.018, which compares to 0.012 calculated assuming independence. The average standard error of the estimated betas for the four portfolios in this period was also 0.018. The average standard errors of the betas for the portfolios of 100 securities in the four subsequent seven-year periods ending June 1961 were respectively 0.025, 0.027, 0.024, and 0.027. Although these standard errors, not assuming independence, are about 50 per cent larger than before, they are still extremely small compared to the range of possible values for portfolio betas.

For the moment, let us therefore assume that the portfolio betas as estimated in the grouping period before adjustment for order bias are extremely reliable numbers in that whatever they measure, they measure it accurately. In this case, adjusting these portfolio betas for the order bias will give extremely reliable and unbiased estimates of the underlying portfolio beta and therefore these adjusted betas can be taken as very good approximations to the underlying, but unknown, values. The greater the number of securities in the portfolio, the better the approximation will be.

The numerical example in Table 2 gives an intuitive feel for what is happening. Consider a portfolio of a large number of securities whose estimated betas were all 0.8 in a particular sample. It will be recalled that such an estimate requires that the true beta be either 0.8 or 1.0. As the number of securities with estimates of 0.8 increases, one can be more and more confident that 75 per cent of the securities have true betas of 0.8 and 25 per cent have true betas of 1.0 or equivalently that an equally-weighted portfolio of these securities has a beta of 0.85.

The heuristic argument in the prior section might lead some to believe that, contrary to the estimates in the grouping period, there are no order biases associated with the portfolio betas estimated in the subsequent seven years. This belief, however, is not correct. Formally, the portfolios formed in the grouping period are being treated as if they were securities in the subsequent period. To estimate these portfolio betas, portfolio returns were calculated and regressed upon some measure of the market. In this paper so far, these portfolio returns were calculated under an equally-weighted monthly revision strategy in which delisted securities were sold at the last available price and the proceeds reinvested equally in the remaining. Other strategies are, of course, possible.

Since these portfolios are being treated as securities, formula (3) applies, so that there is still some "order bias" present. However, in determining the rate of regression, the appropriate measure of the variance of the errors in the estimates is the variance for the portfolio betas and not for the betas of individual stocks. This fact has the important effect of making the ratio of $\sigma^2(\beta_{it})$ to $\sigma^2(\hat{\beta}_{it})$ much closer to one than for individual securities. Estimating $\sigma^2(\hat{\beta}_{it})$ and $\sigma^2(\eta_{it})$ for the portfolios formed on the immediately prior period, the value of this ratio for each of the four seven-year periods from 1933 to 1961 was in excess of 0.99 and for the last seven-year period in excess of 0.98. Thus, for most purposes, little error is introduced by assuming that these estimated portfolio betas contain no "order bias" or equivalently that these estimates measure accurately the true portfolio beta.

A comparison of the portfolio betas in the grouping period, even after adjusting for the order bias, to the corresponding betas in the immediately subsequent period discloses a definite regression tendency. This regression tendency is statistically significant at the five per cent level for each of the last three grouping periods, 1940-47, 1947-54, 1954-61.[12] Thus, this evidence strongly suggests that there is a substantial tendency for the underlying values of beta to regress towards the mean over time. Yet, it could be argued that this test is suspect because the formula used in adjusting for the order bias was developed under the assumption that the distributions of beta were normal. This assumption is certainly not strictly correct and it is not clear how sensitive the adjustment is to violations of this assumption.

A more robust way to demonstrate the existence of a true regression tendency is based upon the observation that the portfolio betas estimated in the period immediately subsequent to the grouping period are measured with negligible error and bias. These estimated portfolio betas can be compared to betas for the same portfolios estimated in the second seven years subsequent to the grouping period. These betas, which have been estimated in the second subsequent period and are given in Table 3, disclose again an obvious regression tendency. This tendency is significant at the five per cent level for the last three of the four possible comparisons.[13]

IV. Summary

Beginning with a review of the conventional wisdom, the paper showed that estimated beta coefficients tend to regress towards the grand mean of all betas over time. The next section presented two kinds of empirical analyses which showed that part of this observed regression tendency represented real nonstationarities in the betas of individual securities and that the so-called order bias was not of overwhelming importance.

In other words, companies of extreme risk—either high or low—tend to have less extreme risk characteristics over time. There are two logical

12. This test of significance was based upon the regression $(\hat{\beta}_{it+1} - 1) = b(\hat{\beta}_{it} - 1) + \epsilon_{it}$ where $\hat{\beta}_{it}$ has been adjusted for order bias. The estimated coefficients with the t-value measured from 1.0 in parentheses were for the five seven-years chronologically 0.86 (−1.14), 0.94 (−0.88), 0.71 (−3.84), 0.86 (−3.23), and 0.81 (−2.57). Note that even if β_{it} were measured with substantial independent error contrary to fact, the estimated b would not be biased towards zero because, as footnote 10 shows, the adjustment for the order bias has already corrected for this bias.

13. Using the same regression as in the previous footnote, the estimated coefficient b with the t-value measured from 1.0 in parentheses were for the four possible comparisons in chronological order 0.92 (−0.69), 0.74 (−2.67), 0.62 (−6.86), and 0.58 (−5.51).

explanations. First, the risk of existing projects may tend to become less extreme over time. This explanation may be plausible for high risk firms, but it would not seem applicable to low risk firms. Second, new projects taken on by firms may tend to have less extreme risk characteristics than existing projects. If this second explanation is correct, it is interesting to speculate on the reasons. For instance, is it a management decision or do limitations on the availability of profitable projects of extreme risk tend to cause the riskiness of firms to regress towards the grand mean over time? Though one could continue to speculate on the forces underlying this tendency of risk—as measured by beta coefficients—to regress towards the grand mean over time, it remains for future research to determine the explicit reasons.

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