University of California, Los Angeles
Department of Statistics

Statistics C183/C283 **Instructor: Nicolas Christou**

Practice problems

Problem 1
You are given the folowing data on 7 stocks:

Stock i	$\bar{R}_i$	σ_i
1	0.15	0.10
2	0.20	0.15
3	0.18	0.20
4	0.12	0.10
5	0.10	0.05
6	0.14	0.10
7	0.16	0.20

Using these data we ranked the stocks based on the excess return to standard deviation ratio ($\frac{\bar{R}_i - R_f}{\sigma_i}$) to compute the entries in the next table. Assume $R_f = 5\%$ and that the average correlation coefficient is $\rho = 0.50$.

Stock i	$\frac{R_i - R_f}{\sigma_i}$	$\frac{\rho}{1-\rho+i\rho}$	$\sum_{j=1}^{i} \frac{R_j - R_f}{\sigma_j}$	C_i
1	1.00	0.5000	1.00	0.5000
2	1.00	0.3333	2.00	0.6667
5	1.00	0.2500	3.00	0.7500
6	0.90	0.2000	3.90	???
4	0.70	0.1667	4.60	0.7668
3	0.65	0.1429	5.25	0.7502
7	0.55	0.1250	5.80	0.7250

a. Find the missing value C_4.

b. What is the composition of the optimum portfolio assuming no short sales?

c. What is the expected return and standard deviation of the combination of the optimum portfolio with the risk free asset (80% and 20%)? Show this combination on the graph of expected return against standard deviation.

Problem 2
You are given the following data:

Stock i	$\bar{R}_i$	σ_i
1	0.29	0.03
2	0.19	0.02
3	0.08	0.15

a. Assume short sales are allowed, $R_f = 0.05$, and $\bar{\rho} = 0.5$. Rank the stocks based on the excess return to standard deviation ratio, find the cut-off point C^*, and find the optimum portfolio.

b. The above solution could have been found using the techniques that discussed earlier in class through the following:

$$\boldsymbol{Z} = \boldsymbol{\Sigma}^{-1}\boldsymbol{R} = \begin{pmatrix} 0.00090 & 0.00030 & 0.00225 \\ 0.00030 & 0.00040 & 0.00150 \\ 0.00225 & 0.00150 & 0.02250 \end{pmatrix}^{-1} \begin{pmatrix} 0.29 - 0.05 \\ 0.19 - 0.05 \\ 0.08 - 0.05 \end{pmatrix} = \begin{pmatrix} 280.00 \\ 320.00 \\ -48.00 \end{pmatrix}.$$

Explain what you see here and verify that the solution is the same as with part (a).

Problem 3
Suppose the single index model holds. Also, short sales are allowed and there is a risk free rate $R_f = 0.002$. For 3 stocks the following were obtained based on monthly returns for a period of 5 years:

Stock i	α	β	σ^2_ϵ
1	0.01	1.08	0.003
2	0.04	0.80	0.006
3	0.08	1.22	0.001

The expected return and variance of the market are $\bar{R}_m = 0.10$ and $\sigma^2_m = 0.002$ for the same period.

a. Suppose that the optimum portfolio consists of 30% of stock 1, 50% of stock 2, and 20% of stock 3. What is the β of this portfolio.

b. Suppose that you are a portfolio manager and you have \$500000 to invest in this optimum portfolio on behalf of a client. In addition this client wants to invest another \$300000 by borrowing this amount at the risk free rate $R_f = 0.002$. What is the expected return and standard deviation of this portfolio. Show it on the expected return standard deviation space.

c. What is the covariance between the portfolio of part (a) and the market?

d. If the client wants to allocate 60% of his initial funds in the optimum portfolio and the remaining 40% in the risk free asset, what would be the expected return and standard deviation of this position?

e. What is the covariance between stock 1 and the market?

Problem 4
Assume that $\sigma^2_m = 10, R_f = 0.05$. You are also given $\beta_1 = 1, \beta_2 = 1.5, \beta_3 = 1, \beta_4 = 2, \beta_5 = 1, \beta_6 = 1.5, \beta_7 = 2, \beta_8 = 0.8, \beta_9 = 1, \beta_{10} = 0.6$. The table below shoes the procedure for finding the cut-off point C^*.

Stock i	$\frac{\bar{R}_i - R_f}{\beta_i}$	$\frac{(\bar{R}_i - R_f)\beta_i}{\sigma^2_{\epsilon i}}$	$\sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\beta_j}{\sigma^2_{\epsilon j}}$	$\frac{\beta^2_i}{\sigma^2_{\epsilon i}}$	$\sum_{j=1}^{i} \frac{\beta^2_j}{\sigma^2_{\epsilon j}}$	C_i
1	10.0	0.20	(a)	0.02000	0.02000	1.67
2	8.0	0.45	0.65	0.05625	0.07625	3.69
3	7.0	0.35	1.00	0.05000	0.12625	4.42
4	6.0	2.40	3.40	0.40000	0.52625	5.43
5	6.0	0.15	3.55	0.02500	0.55125	(c)
6	4.0	0.30	3.85	0.07500	0.62625	5.30
7	3.0	0.30	4.15	0.10000	(b)	5.02
8	2.5	0.10	4.25	0.04000	0.76625	4.91
9	2.0	0.10	4.35	0.05000	0.81625	4.75
10	1.0	0.06	4.41	0.06000	0.87625	4.52

a. Find the three missing values, (a), (b), (c) in the table above.

b. If short sales are not allowed find the cut-off point C^* and the value of z_1.

c. If short sales are allowed find the cut-off point C^* and the value of z_1.

d. Find the correlation coefficient between stock 1 and the market.

Problem 5
For three stocks you are given the following data based on the single index model:

Stock	$\bar{R}$	β	σ^2_ϵ
A	0.0051	0.94	0.0033
B	0.0120	0.61	0.0038
C	0.0160	1.12	0.0046

Below you are given the solution to the problem (the point of tangency) when short sales are allowed and $R_f = 0.005$.

$$\boldsymbol{Z} = \boldsymbol{\Sigma^{-1} R} = \begin{pmatrix} 0.00489048 & 0.00103212 & 0.00189504 \\ 0.00103212 & 0.00446978 & 0.00122976 \\ 0.00189504 & 0.00122976 & 0.00685792 \end{pmatrix}^{-1} \begin{pmatrix} 0.0051 - 0.005 \\ 0.0120 - 0.005 \\ 0.0160 - 0.005 \end{pmatrix} = \begin{pmatrix} -0.883563202 \\ 1.327096101 \\ 1.610164293 \end{pmatrix}.$$

The sum of the z_i's is $\sum_{i=1}^{3} z_i = 2.053697192$ and therefore the x_i's are:
$x_1 = -0.4302, x_2 = 0.6462, x_3 = 0.7840$.

The above is one way to solve the problem. We can also solve the problem by ranking the stocks based on the excess return to beta ratio.

a. Verify the entries in the table below that will allow you to find the C^*. You will also need $\sigma_m^2 = 0.0018$.

Stock i	α_i	$\hat{\beta}_i$	$\bar{R}_i$	$\sigma^{\hat{2}}_{\epsilon i}$	$\frac{R_i - R_f}{\hat{\beta}_i}$	$\frac{(\bar{R}_i - R_f)\hat{\beta}_i}{\sigma^{\hat{2}}_{\epsilon i}}$	$\sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\hat{\beta}_j}{\sigma^{\hat{2}}_{\epsilon j}}$	$\frac{\hat{\beta}_i^2}{\sigma^{\hat{2}}_{\epsilon i}}$	$\sum_{j=1}^{i} \frac{\hat{\beta}_j^2}{\sigma^{\hat{2}}_{\epsilon j}}$	C_i
2	0.0059	0.61	0.0120	0.0038	0.0114754098	1.12368421	1.123684	97.92105	97.92105	0.001719548
3	0.0048	1.12	0.0160	0.0046	0.0098214286	2.67826087	3.801945	272.69565	370.61670	0.004105009
1	-0.0043	0.94	0.0051	70.0033	0.0001063830	0.02848485	3.830430	267.75758	638.37428	0.003208254

b. Assume short sales are allowed. Find C^* and use it to find the composition of the optimum portfolio (point of tangency). Your answer should be exactly the same as above.

c. Assume short sales are not allowed. Find C^* and use it to find the composition of the optimum portfolio.

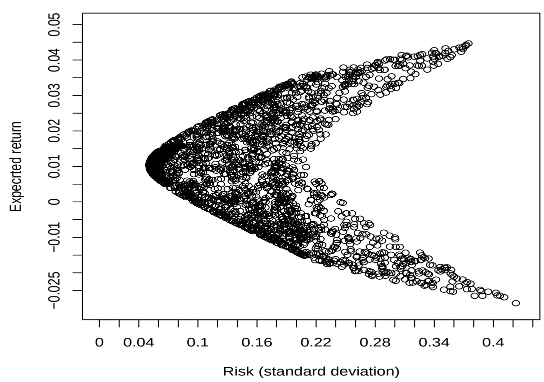
d. Compute the mean return and standard deviation of the portfolios in (b) and (c) and place them (approximately) on the graphs below. Your answer should be the point of tangency in both cases. Note: The first graph allows short sales, while the second graph does not.

e. Write down the expression in matrix form that computes the covariance between the portfolio of part (b) and the equally allocated portfolio $(\frac{1}{3}A, \frac{1}{3}B, \frac{1}{3}C)$. No calculations, just the expression!

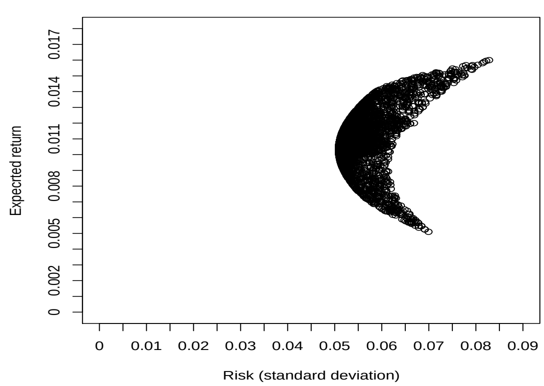
f. Consider the portfolio of part (b). Suppose that you want to place 60% of your funds in portfolio (b) and invest the other 40% in the risk free asset. Find the mean return and standard deviation of this new portfolio and show it on the first graph.

g. You have \$2000 to invest in portfolio (b). In addition you borrow another \$1000 to invest in portfolio (b). Show the position of this portfolio on the first graph (approximately). No calculations.

Short sales allowed (question (d)):



Short sales not allowed (question (d)):



Problem 6
Using the constant correlation model we completed the table below on 12 stocks. Assume $R_f = 0.05$ and average correlation $\rho = 0.45$.

Stock i	$\bar{R}_i$	σ_i	$\frac{R_i - R_f}{\sigma_i}$	$\frac{\rho}{1-\rho+i\rho}$	$\sum_{j=1}^{i} \frac{R_j - R_f}{\sigma_j}$	C_i
1	0.27	0.031	7.097	0.450	7.097	3.194
2	0.31	0.042	6.190	0.310	13.287	4.124
3	0.16	0.023	4.783	0.237	18.070	4.280
4	0.15	0.021	4.762	0.191	22.832	4.372
5	0.33	0.059	4.746	$a =?$	$b =?$	$c =?$
6	0.27	0.061	3.607	0.138	31.184	4.318
7	0.19	0.039	3.590	0.122	34.774	4.229
8	0.13	0.029	2.759	0.108	37.532	4.070
9	0.16	0.051	2.157	0.098	39.689	3.883
10	0.12	0.038	1.842	0.089	41.531	3.701
11	0.08	0.022	1.364	70.082	42.895	3.510
12	0.06	0.028	0.357	0.076	43.252	3.271

a. Find the three missing numbers a, b, c in the table above.

b. Find the cut-off point C^* if short sales are not allowed.

c. Find the cut-off point C^* if short sales are allowed.

d. Write down the expression in matrix form that computes the variance of the portfolio when short sales are allowed. No calculations.

e. You are given a new stock with $\bar{R} = 0.055, \sigma = 0.025$. What will change when short sales are allowed and when short sales are not allowed in terms of the portfolio allocation. Briefly explain your answer without doing all the calculations.

Problem 7
Answer the following questions:

a. Assume that the variance of security A is 0.16 and the variance of security B is 0.25. The variance of a portfolio consisting of $50\%A$ and $50\%B$ is 0.0525. What is the covariance between securities A and B?

b. Assume $R_f = 0.05$ and two stocks A, B with $\bar{R}_A = 0.14, \bar{R}_B = 0.10$. Suppose the point of tangency to the efficient frontier (the one constructed using the two stocks), consists of $60\%A$ and $40\%B$. Let's say that you want to build a portfolio by combining the risk free asset and portfolio G to obtain expected return 11%. Determine the percentages of your investment in the risk free asset and in portfolio G.

c. Consider the data from part (b). Suppose you want to build a portfolio with expected return 0.10 by combining the risk free asset and portfolio G. What is the composition of this portfolio in the risk free asset, in A, and in B?

d. Suppose two stocks have the following: $\bar{R}_A = 0.14, \sigma_A = 0.06, \bar{R}_B = 0.08, \sigma_B = 0.03$. What value of ρ_{AB} will force you to invest everything in the least risky asset?

e. Consider the data from part (d). If short sales are allowed, what composition of A and B will minimize the risk of the resulting portfolio if $\rho_{AB} = 0.80$?

Problem 8
Explain how you would trace out the efficient frontier using the data of problem 3 when short sales are allowed but no riskless lending and borrowing. You do not need to perform any calculations, but you must show a graph and the inputs of the method you are using.