University of California, Los Angeles
Department of Statistics

Statistics C183/C283 **Instructor: Nicolas Christou**

Single index model
Useful formulas

The single index model states that

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it}$$

where, R_{it} is the return of stock i at time t and R_{mt} is the return of the market at time t. Assumptions and notation:

$$E(\epsilon_i) = 0,\ var(\epsilon_i) = \sigma^2_{\epsilon_i},\ E(\epsilon_i \epsilon_j) = 0, cov(R_m, \epsilon_i) = 0, var(R_m) = \sigma^2_m, E(R_m) = \bar{R}_m.$$

Therefore,

$$\begin{aligned} E(R_i) &= \alpha_i + \beta_i \bar{R}_m \\ var(R_i) = \sigma^2_i &= \beta^2_i \sigma^2_m + \sigma^2_{\epsilon_i} \\ cov(R_i, R_j) = \sigma_{ij} &= \beta_i \beta_j \sigma^2_m \end{aligned}$$

Here are some useful formulas:

a. Estimate of β_i (beta of stock i):

$$\hat{\beta}_i = \frac{\sum_{t=1}^m (R_{it} - \bar{R}_i)(R_{mt} - \bar{R}_m)}{\sum_{t=1}^m (R_{mt} - \bar{R}_m)^2}.$$

b. Estimate of α_i (alpha of stock i):

$$\hat{\alpha}_i = \bar{R}_i - \hat{\beta}_i \bar{R}_m.$$

c. Estimate of $\sigma^2_{\epsilon_i}$ (variance of random error term associated with stock i):

$$\hat{\sigma}^2_{\epsilon_i} = \frac{\sum_{t=1}^m e^2_{it}}{m-2} = \frac{\sum_{t=1}^m (R_{it} - \hat{\alpha}_i - \hat{\beta}_i R_{mt})^2}{m-2}.$$

d. Estimate of $var(\hat{\beta}_i)$.

$$var(\hat{\beta}_i) = \frac{\hat{\sigma}^2_{\epsilon_i}}{\sum_{t=1}^m (R_{mt} - \bar{R}_m)^2}.$$

e. Correlation between stock i and stock j:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} = \frac{\beta_i \beta_j \sigma^2_m}{\sigma_i \sigma_j}.$$

f. Correlation between stock i and market:

$$\rho_{im} = \beta_i \frac{\sigma_m}{\sigma_i} \Rightarrow \beta_i = \rho_{im} \frac{\sigma_i}{\sigma_m}.$$

Simple R commands:

```
a1 <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/
stocks5_period1.txt", header=TRUE)

#Regression of r11 on rsp1 (index):
q <- lm(a1$r11 ~ a1$rsp1)

#Summary of the regression above:
summary(q)

#List the names of the results in object q:
names(q)

#Get the estimates of alpha and beta:
q$coefficients[1]
q$coefficients[2]

#List the residuals:
q$residuals

#Get the estimate of the variance of the error term (MSE):
sum(q$residuals^2)/(nrow(a1)-2)

#Another way:
summary(q)$sigma^2

#variance-covariance matrix of the estimates of the main parameters
#of the model:
vcov(q)

#Get the variance of the estimate of beta:
vcov(q)[2,2]

#Another way:
summary(q)$coefficients[4]^2
```