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Statistics C183/C283 **Instructor: Nicolas Christou**

Single index model - example

For three stocks you are given the following data based on the single index model:

Stock	α	β	σ^2_ϵ
A	-0.0043	0.94	0.0033
B	0.0059	0.61	0.0038
C	0.0048	1.12	0.0046

Assume $\bar{R}m = 0.01$ and $\sigma^2_m = 0.0018$.

Below you are given the solution to the problem (point of tangency) when short sales are allowed and $R_f = 0.005$ using two methods:

A. Using the formula $\boldsymbol{Z} = \boldsymbol{\Sigma^{-1}R}$:

$$\boldsymbol{Z} = \boldsymbol{\Sigma^{-1}R} = \begin{pmatrix} 0.00489048 & 0.00103212 & 0.00189504 \\ 0.00103212 & 0.00446978 & 0.00122976 \\ 0.00189504 & 0.00122976 & 0.00685792 \end{pmatrix}^{-1} \begin{pmatrix} 0.0051 - 0.005 \\ 0.0120 - 0.005 \\ 0.0160 - 0.005 \end{pmatrix} = \begin{pmatrix} -0.883563202 \\ 1.327096101 \\ 1.610164293 \end{pmatrix}.$$

The sum of the z_i's is $\sum_{i=1}^3 z_i = 2.053697192$ and therefore the x_i's are:
$x_1 = -0.4302, x_2 = 0.6462, x_3 = 0.7840.$

B. Using the single index model: Ranking the stocks based on the excess return to beta ratio.

Stock i	α_i	$\hat{\beta}_i$	$\bar{R}_i$	$\sigma^{\hat{2}}_{\epsilon i}$	$\frac{R_i - R_f}{\hat{\beta}_i}$	$\frac{(\bar{R}_i - R_f)\hat{\beta}_i}{\sigma^{\hat{2}}_{\epsilon i}}$	$\sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\hat{\beta}_j}{\sigma^{\hat{2}}_{\epsilon j}}$	$\frac{\hat{\beta}^2_i}{\sigma^{\hat{2}}_{\epsilon i}}$	$\sum_{j=1}^{i} \frac{\hat{\beta}^2_j}{\sigma^{\hat{2}}_{\epsilon j}}$	C_i
2	0.0059	0.61	0.0120	0.0038	0.0114754098	1.12368421	1.123684	97.92105	97.92105	0.001719548
3	0.0048	1.12	0.0160	0.0046	0.0098214286	2.67826087	3.801945	272.69565	370.61670	0.004105009
1	-0.0043	0.94	0.0051	0.0033	0.0001063830	0.02848485	3.830430	267.75758	638.37428	0.003208254

From the table above we get $C^* = 0.003208254$ (short sales are allowed therefore C^* is the last C_i. Using,

$$z_i = \frac{\beta_i}{\sigma^2_{\epsilon i}} \left(\frac{\bar{R}_i - R_f}{\beta_i} - C^* \right)$$

we compute the $z_i's$. We get:

$$z_1 = \frac{0.94}{0.0033}[0.0001063830 - 0.003208254] = -0.8835632$$

$$z_2 = \frac{0.61}{0.0038}[0.0114754098 - 0.003208254] = 1.3270961$$

$$z3 = \frac{0.1.12}{0.0046}[0.0098214286 - 0.003208254] = 1.610164$$

The sum of the $z_i's$ is:

$$\sum_{i=1}^{3} z_i = 2.053697, \text{ and therefore}$$

using

$$x_i = \frac{z_i}{\sum_{i=1}^n z_i} \text{ we compute the } x_i's:$$

$x_1 = \frac{-0.8835632}{2.053697} = -0.4302305.$

$x_2 = \frac{1.3270961}{2.053697} = 0.6461985.$

$x_3 = \frac{1.610164}{2.053697} = 0.7840320.$

The two methods give exactly the same answer.