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Statistics C183/C283 **Instructor: Nicolas Christou**

Trace out the efficient frontier (similar to Mutual Fund Theorem)
Short sales allowed
No riskless lending and borrowing
Example

On the next page we can see many combinations of three stocks (short sales are allowed). The characteristics of the stocks are:

Stock	$\bar{R}$	σ^2
1	0.14	0.0036
2	0.08	0.0064
3	0.20	0.0400

The correlation coefficients are: $\rho_{12} = 0.5, \rho_{13} = 0.2, \rho_{23} = 0.4$.

We assume the existence of two risk free rates to trace out the entire efficient frontier. Let $R_{f1} = 0.05$, and $R_{f2} = 0.08$. We will find the point of tangency for each one of the two risk free rates (points A and B).

- We begin with $R_{f1} = 0.05$ to find point A. We need to compute the z_i's first:

$$\boldsymbol{Z_A} = \boldsymbol{\Sigma^{-1} R_1} = \begin{pmatrix} 0.0036 & 0.0024 & 0.0024 \\ 0.0024 & 0.0064 & 0.0064 \\ 0.0024 & 0.0064 & 0.0400 \end{pmatrix}^{-1} \begin{pmatrix} 0.14 - 0.05 \\ 0.08 - 0.05 \\ 0.2 - 0.05 \end{pmatrix} = \begin{pmatrix} 29.166667 \\ -9.821429 \\ 3.571429 \end{pmatrix}.$$

  The sum of the z_i's is $\sum_{i=1}^{3} z_i = 22.917$.
  Therefore $x_1 = \frac{29.166667}{22.917} = 1.2727, x_2 = \frac{-9.821429}{22.917} = -0.4286, x_3 = \frac{3.571429}{22.917} = 0.1558$.
  Compute the mean and variance of the point of tangency A:

$$\bar{R}_A = \boldsymbol{x'_A \bar{R}} = 0.1751.$$

$$\sigma_A^2 = \boldsymbol{x'_A \Sigma x_A} = 0.005457.$$

- Now we find the point of tangency (point B) when $R_{f2} = 0.08$: We need to compute the z_i's first:

$$\boldsymbol{Z_B} = \boldsymbol{\Sigma^{-1} R_2} = \begin{pmatrix} 0.0036 & 0.0024 & 0.0024 \\ 0.0024 & 0.0064 & 0.0064 \\ 0.0024 & 0.0064 & 0.0400 \end{pmatrix}^{-1} \begin{pmatrix} 0.14 - 0.08 \\ 0.08 - 0.08 \\ 0.2 - 0.08 \end{pmatrix} = \begin{pmatrix} 22.222222 \\ -11.904762 \\ 3.571429 \end{pmatrix}.$$

  The sum of the z_i's is $\sum_{i=1}^{3} z_i = 13.889$.
  Therefore $x_1 = \frac{22.222222}{13.889} = 1.60, x_2 = \frac{-11.904762}{13.889} = -0.8571, x_3 = \frac{3.571429}{13.889} = 0.2571$.
  Compute the mean and variance of the point of tangency B:

$$\bar{R}_B = \boldsymbol{x'_B \bar{R}} = 0.2069.$$

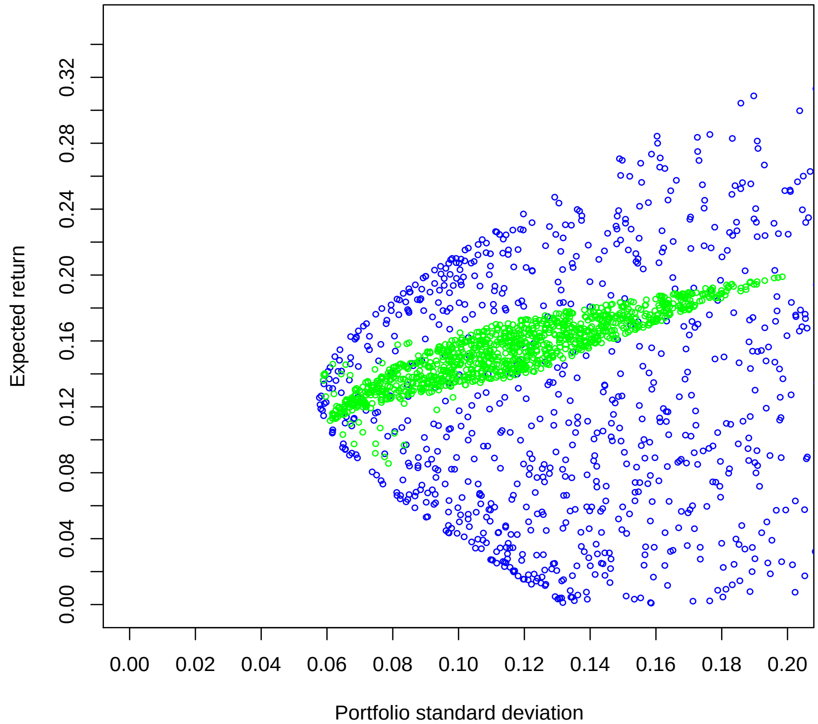
$$\sigma_B^2 = \boldsymbol{x'_B \Sigma x_B} = 0.009134.$$

- We also need the covariance between portfolios A and B:

$$\sigma_{AB} = \boldsymbol{x'_A \Sigma x_B} = 0.006845.$$

- We treat now portfolios A and B as two "stocks". Since we know their mean returns, variances, and covariance we can choose many combinations (allowing short sales) to trace the entire efficient frontier. This is shown on the last page.

The plot of many portfolios of the three stocks:



Trace out the efficient frontier: The plot below was constructed using many combinations of portfolios A and B allowing short sales.