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Statistics C183/C283 **Instructor: Nicolas Christou**

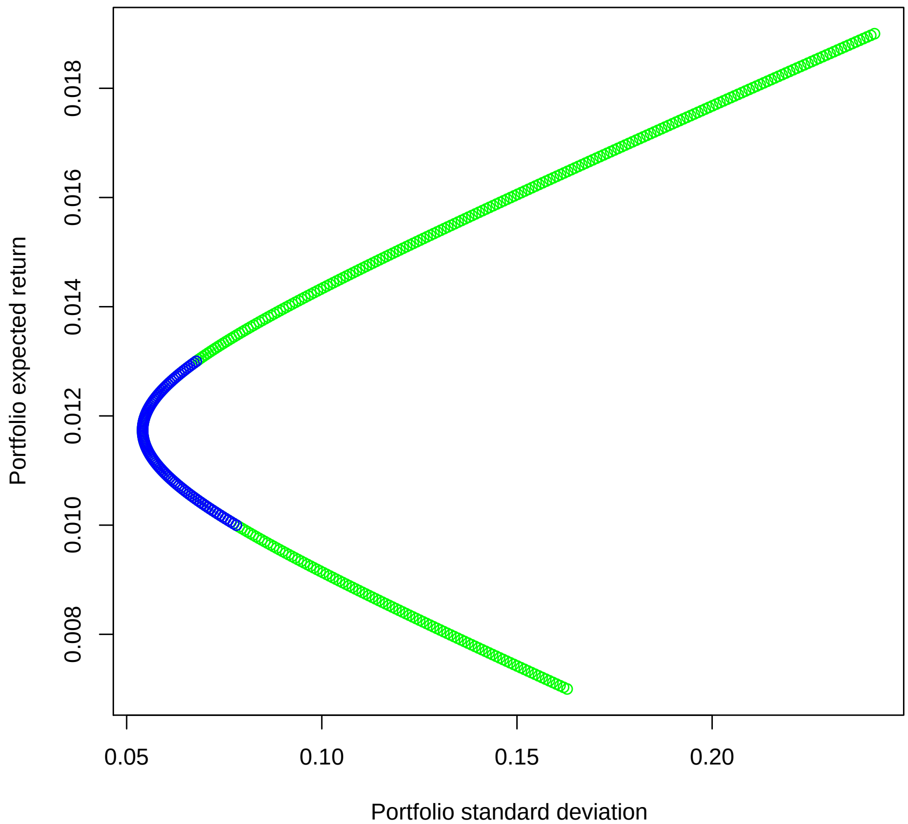
Portfolio possibilities curve
Efficient frontier
Example using R

Means, variances, covariance are given:
Suppose two stocks have the following expected return and variance: $\bar{R}_A = 0.01, \bar{R}_B = 0.013, \sigma_A^2 = 0.061, \sigma_B^2 = 0.0046$, and $\sigma_{AB} = 0.00062$.

The following simple R commands will read these inputs and construct the portfolio possibilities curve with and without short sales allowed.

```
#Read expected returns, variances, and covariances:
r1_bar <-  0.01
r2_bar <-  0.013
var1 <-  0.0061
var2 <-  0.0046
cov_12 <-  0.00062

#Short sales NOT allowed:
a <- seq(0, 1, 0.01)
b <- 1-a

rp_bar_ns <- a*r1_bar+b*r2_bar
var_p_ns <- a^2*var1+b^2*var2+2*a*b*cov_12
sd_p_ns <- var_p_ns^0.5


#Short sales allowed:
a1 <- seq(-2,2, 0.01)
b1 <- 1-a1

rp_bar_s <- a1*r1_bar+b1*r2_bar
var_p_s <- a1^2*var1+b1^2*var2+2*a1*b1*cov_12
sd_p_s <- var_p_s^0.5

#Plot the two portfolio possibilities curves:
#Green:  Short sales allowed.
#Blue:  Short sales not allowed.
plot(sd_p_s, rp_bar_s, col="green", xlab="Portfolio standard deviation",
     ylab="Portfolio expected return")

points(sd_p_ns, rp_bar_ns, col="blue")
```

Portfolio possibilities curve:



Read data from a .csv file from http://finance.yahoo.com:
Get data on two stocks from `http://finance.yahoo.com` and save on your computer two `.csv` files, named `table1.csv` and `table2.csv`. We read the `Close` prices, convert them into returns, compute the mean return and variance for each stock, the covariance between the two stocks, and do the rest of the analysis. Here are some simple commands in `R`:

```
#Read the two files:
a1 <- read.table("table_1.csv", header=TRUE, sep=",")
a2 <- read.table("table_2.csv", header=T, sep=",")

#Or access from the course website:
a1 <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/
      table_1.csv", header=TRUE, sep=",")

a2 <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/
      table_2.csv", header=TRUE, sep=",")

#Select only the close prices from each file:
prices <- as.data.frame(cbind(a1$Close, a2$Close))

class(prices)

#You can rename the Close prices as follows:
names(prices) <-c("p1", "p2")

#You can see your data:
prices$p1
prices$p2

#Convert the close prices into returns:
r1 <- (prices$p1[-length(prices$p1)]-prices$p1[-1])/prices$p1[-1]
r2 <- (prices$p2[-length(prices$p2)]-prices$p2[-1])/prices$p2[-1]

#Place the returns together in a data frame:
returns <- as.data.frame(cbind(r1,r2))

#Construct a histogram of the returns:
hist(r1)
hist(r2)

#The following 5 lines will save the histograms in a pdf file on your
#current working directory:
pdf("hist_r1_r2.pdf", h=3.0)
par(mfrow=c(1,2))
hist(r1, main="Histogram 1", xlab="Stock1")
hist(r2, main="Histogram2", xlab="Stock2")
dev.off()
```

```
#Get summary statistics of the returns:
summary(returns)

#Get the mean returns:
mean(returns)

#Get the variance covariance matrix of the returns:
cov(returns)

#Create many portfolios (combinations of the two stocks):
a <- seq(0,1,.01)
b <- seq(1,0,-.01)
#Or simply
b <- 1-a

#Compute the expected return of each portfolio:
rp_bar <- a*mean(returns$r1)+b*mean(returns$r2)

#Compute the variance and standard deviation of each portfolio:
var_p <- a^2*var(returns$r1)+b^2*var(returns$r2)+
         2*a*b*cov(returns$r1,returns$r2)

sd_p <- var_p^.5

#Create a data frame of the standard deviation and expected return of
#each portfolio:
qq <- as.data.frame(cbind(sd_p, rp_bar))

#Plot the portfolio possibilities curve:
plot(sd_p, rp_bar, xlab="Portfolio risk (standard deviation)",
     ylab="Expected return")

#You can get the same plot by simply doing this:
plot(qq, xlab="Portfolio risk (standard deviation)",
     ylab="Expected return")

#If you want a line instead of points:
plot(qq, xlab="Portfolio risk (standard deviation)",
     ylab="Expected return", type="l")

#Find the composition of the minimum risk portfolio:
x1 <- (var(returns$r2)-cov(returns$r1,returns$r2))/
      (var(returns$r1)+var(returns$r2)-2*cov(returns$r1,returns$r2))
x2 <- 1-x1
```

```
#Compute the expected return and standard deviation of the minimum risk
#portfolio:
rp_bar_min <- x1*mean(returns$r1)+x2*mean(returns$r2)

sd_p_min <- (x1^2*var(returns$r1)+x2^2*var(returns$r2)+
             2*x1*x2*cov(returns$r1,returns$r2))^0.5

#Add the minimum risk portfolio on the previous plot:
points(sd_p_min, rp_bar_min, col="green", pch=19)

#Identify the efficient frontier.
#First find the portfolios that have expected return above the expected return
#of the minimum risk portfolio:
qqq <- qq[qq$rp_bar > rp_bar_min, ]

#And then draw the efficient frontier:
points(qqq, col="blue", type="l", lwd=5)

#Save the graph in a pdf file:
pdf("portfolio_curve_s10.pdf")
plot(qq, xlab="Portfolio risk (standard deviation)",
     ylab="Portfolio expected return", type="l")
points(sd_p_min,rp_bar_min, col="green", pch=19)
points(qqq, col="blue", type="l", lwd=5)
dev.off()
```

Here is the plot: