Quiz 5 Solution

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If X is a binomial random variable with parameters n and p, then we have calculated that E(X) = np and Var(X) = np(1-p).

Often, the value of p is unknown and we are interested in estimating p. For example, p might be the probability that a randomly selected person will support Grey Davis for Governor. Or it might be the probability that a baseball player will get a hit. Or it might be the probability that a suspicious looking coin will actually land "heads".

A good estimate of p is X/n, that is, the number of successes (X) divided by the number of trials (n). Let Y = X/n.

a) Find a formula for the expectation of Y.

Using the rules for expectations of linear combinations of random variables:
E(Y) = E(X/n) = (1/n)*E(X) = (1/n) * (np) = p.

b) Find a formula for the standard deviation (a.k.a. the standard error) of Y.

$\mathrm{Var}(Y) = \mathrm{Var}(X/n) = (1/n)^2\ \mathrm{Var}(X) = (1/n)^2\ (n*p*(1-p)) = p*(1-p)/n$

SD(Y) is the square-root of this expression.

c) If n is sufficiently large, the pdf of Y has an approximate normal distribution. Suppose we flip a fair coin 100 times. What's the approximate probability that Y will be bigger than 55%?

E(Y) = p = .50
SD(Y) = sqrt(p*(1-p)/n) = sqrt(.5*.5/100) = .05

So Y 's pdf is approximately N(.50, .05). Therefore P(Y > .55) = P(Z > (.55 - .5)/.05), approximately, and this
= P(Z > 1) = 1 - pnorm(1)
You can get this value without using a table. We know P(-1 < Z < 1) is approximately .68, which means that there is .32 "outside" of this range and, since the pdf is symmetric, there must be .16 below -1 and .16 above 1. Therefore, P(Z > 1) = .16.