Quiz 8 Solutions

Friday, May 31, 2002 Stats 110A Name: ID:

Clara the Psychic Bulldog claims to be able to correctly predict the outcome of a cointoss. Not with perfect accuracy mind you (she is, after all, only a dog) but she can do better than chance (or so her owner claims.) In case you were wondering, one bark is "heads", two barks is "tails". A crackshot team of paranormal investigators decides to put her to the test in a trial consisting of 10 flips of a fair coin. Let X represent the number of flips that Clara predicts correctly, and let p represent the probability that she can get the right answer.

1) State the null and alternative hypotheses.

Remember that these hypotheses are statements about parameters. The appropriate parameter here is p -- the probability of getting the right answer on a single toss. Another apropriate choice might be mu = np = number of correct calls out of n tosses. Here are acceptable pairs:

H0: p = .5 Ha: p > .5 OR H0: np = 5 Ha: np > 5

Note that the alternative must be ">" since this would be the case if Clara were psychic.

2) Give a test statistic you would like to use for this and state its sampling distribution (assuming the null hypothesis is true.)

You should recognize that this is a binomial situation. A good estimate of p is X/n and a good estimate of np is X. Your test should be based on this. Normal-distribution based tests such as the z-test or t-test are acceptable approximations to the binomial test only if $np \ge 10$ AND $n^*(1-p) \ge 10$, which is not the case here.

The sampling distribution of X/n is binomial with n = 10, p = .5. Same for X.

3) The results are in. Clara got 7 of the 10 flips correct. What is the p-value?

p-value = Probabily of test stat as extreme or more extreme than observed value = $P(X \ge 7)$. It is important that you use greater-than-or-equal-to, since this makes a big difference in this problem.

 $P(X \ge 7) = 1 - P(X < 7) = 1 - P(X <= 6)$. Using the table provided, this is 1 - .8281 = .1719. (Use the rows associated with n = 10 and the column associated with p = .50.)

4) Using a significance level of 5%, what do you conclude about Clara's psychic abilities?

.1719 > .05, therefore we do not reject the null hypothesis. There is no evidence that Clara is psychic.

The attached tables gives probabilities for $P(X \le x)$. If you want, for example, P(X = 4), then this is $P(X \le 4) - P(X \le 3)$.