

QUETELET ON PROBABILITIES

Instructions Populaires sur le Calcul des Probabilités, par A. Quetelet. Bruxelles. Chez H. Tarlier et M. Hayez, Rue de la Montagne, 1828.

IT has before occurred to us to make some observations on the study of the Theory of Probabilities as a part of education. From this we have been hitherto deterred by not finding a book on the subject sufficiently clear for the use of very young persons; but the work of M. Quetelet, which has lately fallen in our way, has shown us that the science can be usefully exhibited in a form which need frighten no one who is master of the common rules of arithmetic. The author is most advantageously known to the scientific world by his statistical works, by the '*Correspondence Mathématique et Physique*,' of which he is editor, and by an elementary collection of the phenomena of natural philosophy, called '*Positions de Physique*.'

Previously to entering upon our account of M. Quetelet's work, we shall make some observations upon the study itself, with a view to show why it is worthy of being made a part of education. If there be one subject more than another to be found in the whole circle of the sciences, whose object and reasonings have been utterly misunderstood, it is the theory of probabilities. The consequence has been, that while some have censured it as profane, others have ridiculed it as a delusion, while most are apt to consider it as useless and unpractical. The first objection has arisen from a notion which, spite of the ignorance of the subject which it betrays, is entitled to a respectful answer. The theory of probabilities or chances is unfortunate in wanting a name which shall at once point to the subjects in which it is conversant. The word probable, in common language, is applied only to things so likely to happen, that the expectation of their occurrence is strong enough to guide our actions; while chance, as opposed to Providence, has been used in a will known sense by those few who have not been able to collect from all they see around them a conviction of the existence of a Creator. It has therefore been imagined by some, that the doctrine of chances is near akin to presumption, if not to atheism,¹ as supposing at the outset that events happen, as the phrase is, by chance. No notion can be more groundless. Our daily determination as to our conduct in the various circumstance of life, are certainly founded upon judgements concerning future events, as to which we can predict nothing absolutely. The reason obviously is, ignorance of the whole state of any case. Nevertheless, experience of preceding events will incline our minds more or less strongly towards the expectation of one or the other occurrence, according as we have seen the same occur more or less often on former occasions. Thus, that a shower of rain will follow a fall in the barometer, is not by many degrees so probable, as that a stone will fall to the ground when its support is removed. Our experience of the latter result is so uniform, that we feel the highest degree of certainty as to any new

¹There are some who are more strongly led by great names than by arguments. To such we would mention, though we do not think it of sufficient importance to dwell upon either side, that the father of this theory, as he is called by Laplace, was PASCAL.

experiment, which is not the case in the former. Nevertheless, the former is no more an accident than the latter, but only depending on circumstances with which we are not so well acquainted. This being the case, how strong soever our conviction may be, that the event about to happen has been pre-arranged by a power with whom everything is certainty, we are not thereby furnished with any rule of conduct, except the notion, that what has happened most often before, in circumstances which appeared to us similar, far from proceeding out of any admission of events happening by chance, is a consequence of the directly opposite belief; for, if preceding occurrences had been purely fortuitous, the arrival of one event would furnish no probability whatever for the repetition of the same under similar circumstances. The order and arrangement of the material universe is usually considered the convincing proof of the existence of a Creator. If so, every investigation which shall establish a general and abiding law, adds some new force to this argument, and every method proceeding out of the law so established, may be made to strengthen those habits of mind, of which such a conclusion should induce the cultivation. The theory of probabilities depends, for much of its practical utility, upon observations which have proved that events, which individually appear to follow no plan, are nevertheless in the mass as much the subjects of a general rule as any others. If the course of human mortality did not continue nearly the same from year to year, no application of the Theory of Probabilities could be made to the insurance of life; and those valuable institutions, which are now among the most secure of all commercial speculations, would either not exist at all, or must charge such a price for the security they give as would render them practically useless. The same might be said of friendly societies, and even of marine and fire insurances. No man accustomed to such considerations can long doubt that the doctrine of chances, instead of being what it has been represented to be, is nothing more than the application of common prudence to these cases, in which it has pleased the Creator to hide from us the arrangement on which he has determined. Our powers of perception and calculation may be employed in as praiseworthy a manner on such an object, as on the construction of a bridge or a canal. It is by no means necessary that an event should be unknown to all, in order to render it a fit subject for the calculation of probability by those who are in ignorance. Let the owner of a lottery be at liberty to distribute prizes and blanks among his contributors at pleasure, and that with every knowledge of their circumstances and motives; yet so long as the arrangements which he follows, and the reasons which guide him, cannot be detected by observation of what he has already done, the theory of probabilities should be as much the guide of those who would speculate without fear of ruin, as if the numbers were to be drawn indiscriminately from the wheel, The word chance is merely an expression of our ignorance of the chain of events which have led to any particular occurrence, and in so strong a light would this be set by the study of the application of the doctrine of chances, that the results would rather tend to correct the notions which the uneducated attach to the word, than to couple it with any opinions contrary to the supreme guidance of an intelligent Creator.

We have heard it objected, that the habit of calculating probabilities may

lead to a passion for gambling. Unfortunately this propensity is so easily acquired by intercourse with the world, that if, by studying the doctrine of chances, those who will game in any case could be prevented from seriously injuring their circumstances, the advantage thus gained would perhaps not be overbalanced by an addition to their number. The habit itself is usually the consequence of the want of ideas and occupations which results from a bad education, and springs more from the desire of excitement, and escape from ennui, than from any reflection upon the possible profit which may accrue from success. It is certain that the desire of gain soon becomes the master passion of a gambler; but we think most will agree with use, that neither the age nor circumstances of the majority of those who commence this pursuit, justify us in supposing that it is their leading motive at the first outset. Be this as it may, it is a common remark, that in proportion as a game is one of skill, the sums usually staked upon it are lessened. Few ever think of gambling at chess; even whist is not commonly a vehicle for high play, while rouge et noir, and games of that class, seem invented only that a few who understand this theory, may make the unthinking world pay dear for the pleasure of a moment's excitement. This would be seriously lessened, if a greater quantity of knowledge, as to the real value of the supposed advantage, could be distributed among the different classes of society. In proportion as any game of hazard is converted into one of skill, the strength of the evil stimulus will be diminished, and its place will be supplied by a more useful excitement,—that of competition for victory only. The fair player would thus be a less easy prey to the sharper, whose very occupation it is, to avail himself of a science which the public rejects, to the ruin of those who are ignorant of it. A cool temperament and great practice in the theory are the necessary requisites of the accomplished thief, who would get his living by play; and the same qualities are necessary to oppose him with success. And be it remembered that the doctrine of chances—a dry arithmetical subject—is rather likely, in common with other pursuits of the same kind, to repress than to create any craving for such excitement as that of gambling.

It is argued by another class of disputants, that the theory, however true in the abstract, can never be applied to practice, inasmuch as we are unacquainted with any way of determining the actual probability of most events, such knowledge being almost as much above our reach as the power of predicting them. This, though perfectly true, is irrelevant, since it is not asserted that the whole conduct of life can be determined by numerical computation. The same argument might be applied, more or less, to all branches of natural philosophy, every one of which is conversant with notions more exact than the data to which it is afterwards to be applied. The advantage of the theory of probabilities, lies in helping the student to form a habit of judging correctly in cases which are beyond the reach of calculation, by accustoming his mind to the consideration of others, in which numerical data and mathematical demonstration can be applied. It is to the understanding, as Laplace has well expressed it, what the sense of touch is to the sight, a corrector of false impressions and a check on premature decisions. Those who know the subject are aware how apt judgement is to be deceived, even in the simplest questions, which, from the definite value

of their circumstances, can be reduced to calculation. Any pursuit which would leave the student with a strong impression of the weakness of his powers, and the fallacy of first impressions, would be a valuable assistant to the teacher; and in this respect we appeal to all who understand the subject, whether there be anything in mathematics or natural philosophy, in which even the proficient is more likely to err, or in which his errors can be more certainly exposed, and when detected stand in a more ridiculous light, than the theory of which we speak. We will now examine its main principle, and see whether it be any more than the application of common arithmetic to a notion already existing in the mind, though in a vague form. It is, however, perfectly capable of precise definition, and when defined, presents results which a little reflection will readily induce us to admit, in all cases so simple, as to be rationally considered as falling under the province of the unassisted judgement.

Suppose a bag to contain four white and two black balls, so placed that we can see no reason why one should be drawn rather than another. Whatever probability the presence of each white ball adds to the chance of a white ball being drawn, the same will each black ball give to the other supposition: we have therefore six events equally possible, four of which are favourable to the production of a white, and two of a black ball. Hence the probability of drawing a white ball is said to be to that of a black one as four to two, or the former is twice as probable as the latter. The fractions $4/6$ and $2/3$ are made to represent these probabilities, the denominator being the whole number of possible cases, and the numerator that number out of them which is favourable to the production of the proposed event. Similarly, more complicated questions admit of an inquiry into the number of ways in which, under given conditions, an event may happen or fail, and the proportion of these two is that of the probabilities for and against its happening. It is only against the preceding illustration that any objections to the theory of probability can be urged; all the rest consists of processes of pure mathematics, admitting of no question. It is a very common mistake to suppose, that there is in mathematics something which gives the first ideas of natural sciences, and creates notions differing entirely in kind from those in common use. This is only true in respect of the accuracy which mathematicians can confer on ideas which, without this science, would have been too vague to furnish subjects of calculation. But in any other sense it is not correct; thus, in the present instance, every one will allow that the chance of drawing a white ball from a bag, which contains a thousand white balls and only one black, is much greater than that of drawing the black one; the step made by the mathematician is simply that of estimating these probabilities by the fractions $1000/1001$ and $1/1001$, against which, if it be objected that a wrong measure of the expectation has been taken, it is all that can be brought; but it must certainly be allowed that the notion is one believed and acted upon, more or less correctly, by every individual in the world. We will now mention some cases in which mankind are subject to err, and where the liability to mistake at least is pointed out, by observation of the common problems of chances.

Most of the arguments of which we make daily use in books or conversations, consist of assertions which are only more or less probable, and whose absolute

certainty there is no method of establishing. The conclusions which are drawn partake of the uncertainty of the premises; so much will be allowed by any one who ever found himself in the wrong. But it is not so generally remembered, that the conclusion of an argument may be improbable, even though following logically from premises, each of which by itself has probability in its favour. The argument, that if A is B, and B is C, A is C, is incontestable; so also is the following: that if there is any probability that A is B, and any probability that B is C, there is some probability that A is C. But it does not follow that if it be more likely than not that A is B, and also that B is C, it is more likely than not that A is C. If there were an even chance for each of the two first, there would be three to one against the conclusion, and there must be more than two to one in favour of each of the premises before the conclusion can be called as likely as not. In a still larger collection of equally probable arguments, still more does any feebleness in the premises affect the likelihood of the conclusion, so that no result which depends on ten equally probable arguments can be considered as having an even chance in its favour, unless there be more than nine to one for each of the premises. Such conclusions are indisputable, in cases where all the circumstances which render an event probable or improbable are known, as in the case of a lottery of black and white balls; and the knowledge thus obtained might be beneficial to a disputant; for though he could not apply numerical calculation to a question of politics or morals, he might thereby be induced to recollect that strong probability, and not certainty, is all that he can hope to arrive at, and might learn caution, both in forming his own opinions, and in condemning those of others. Another common error is, the belief that the occurrence, which, under the circumstances, is most likely to happen, is therefore probable. If a halfpenny be thrown twenty times into the air, the most probable supposition is, that there will come up as many heads as tails—that is, this combination is more likely than any other combination, but not more likely than the arrival of some out of all the other combinations. If, then, a halfpenny were thrown up twenty times, and this were repeated any number of times in succession, we might expect beforehand that there would be a preponderance either of heads or tails in each set, and so it would generally prove. The same would hold of two players of equal skill, who should play different sets of twenty games each. Nevertheless, this result is misunderstood by players in general, and under the name of a run, either of good or bad luck, has many superstitious notions attached to it. It is not recollected, that the result of no one or more games can be considered as in any way likely to influence those which succeed, except so far as they prove superior skill in the party who has won them. It is, we may observe, very common to deduce from an event which has happened, a probability of precisely the contrary nature from that warranted by common sense. Thus a person will say,—I have been robbed to-day, and do not think I shall be robbed again; for it is very unlikely that a man should be robbed twice in one day. Nothing can be more incorrect than this argument: before the robbery happened, certainly the probability of being robbed twice in one day was very much less than that of being robbed once; but after the first event, the second is just as likely as the first was; or more so, if the happening of the first

event be allowed to afford any presumption, however small, against the good management of the person to whom it occurred.

In the Parisian lottery, it is always usual to stake upon a number which has not appeared for some drawings, under the idea that its appearance is rendered more likely by its not having been drawn for some time. Many more such errors might be noticed, even in those subjects which are peculiarly the province of the theory of chances; and still more in the speculations of common life. These serve to show the utility of this science, and, if we are warranted in assuming that the correction of an erroneous habit in thinking of one subject, is likely to exert an influence over our method of treating others, we may recommend it as a part of education.

The work of M. Quetelet is a small and neatly printed duodecimo of 236 pages, written in tolerably easy French, which we hope is not now so great an obstacle to its being put into the hands of a young person, as would formerly have been the case. Each chapter is succeeded by a few questions on its contents. The principles are laid down with great simplicity and correctness, and accompanied by interesting and instructive illustrations. The demonstrations are entirely arithmetical, requiring no more knowledge than that of fractions. We also find in it several things, which it has not hitherto been usual to introduce into popular works on the subject. Among these is the calculation of the moral expectation, 'espérance morale,' in which the sum stated, is considered as having a value dependent on the fortune of the player. The principle employed is the common one, namely, that the values of the sum to two different persons are inversely as the whole possessions of the two. M. Quetelet then proceeds to expose the lottery, as it now stands in France. The constitution of this lottery differs materially from that formerly established in England. There are only ninety numbers, of which five are drawn at once. No one of these is in itself either blank or prize; each candidate for a prize, having previously deposited a certain sum upon the coming up of one number, or of a combination of given numbers, has his stake multiplied by a certain number of times, if the event, on which he has placed his money, happens to arrive. In proof of our previous assertion, that a public well informed on the theory of probabilities would never tolerate the system of lotteries, as it stands at present, we subjoin the meanings of the different hazards, with the sums that may be gained by each, and also those which ought to be gained, if the player were quite even. The stake is supposed to be one franc.

L'extrait simple. Here the player stakes upon a number, named by himself, being one of five. If he wins, he ought to receive 18 francs; but the government gives only 15.

L'extrait déterminé. Here the player names, not only the number, but also which of the five it is to be. His fair gain is 90 francs, and the government gives him no more than 70.

Ambe and Ambe déterminée. Here the player name two numbers instead of one. His fair gain in the first case is $400\frac{1}{2}$ francs, he receives only 270; in the second it is 8010 francs, of which he receives only 5100.

Terne and Quaterne. Here three and four numbers are named. The players

who win these stakes ought to receive 11,748 and 511,038 francs. The government only gives 5500 and 75,000 francs.

With such advantages, it will not surprise the reader to be told, that the French government gains annually more than 160,000 pounds sterling by this very equivocal source of revenue, being 25 per cent. on every sum staked.

The author proceeds to a class of questions, the reasoning of which it is impossible to introduce, from the very complicated nature of the processes; but the results of which are both entertaining and useful. We allude to those cases where an event has been observed to happen a certain number of times, of the previous chances for the happening of which we are in total ignorance. To a person wholly unacquainted with mathematics, it would appear difficult to reduce such questions to calculation, and still more so to one who, though acquainted with the elementary parts of the science, is not well versed in the integral calculus. For example, all the eleven planets which have been yet discovered, move in the same direction round the sun; what degree of probability is there, that if a new planet were discovered, the same thing would be found to hold good? We are here supposed to know no reason why it should be more probable that a planet would move in one direction rather than another. Nearly connected with the same species of questions is the celebrated *method of least squares*, for ascertaining the most probable value, which can be obtained from a set of discordant results. This method has been explained by M. Quetelet, with perhaps as much simplicity as the subject will admit of, to those unacquainted with mathematics. This is followed by a chapter on the law of mortality, and its applications to annuities and insurances, in the most simple cases. M. Quetelet then proceeds to a subject which may excite a smile, when mentioned as one to which numerical analysis can be applied: viz. the probability of the truth of evidence. But it must be recollected, that it is one thing to assert that the probability of any one witness telling the truth *can be found*, and another that the same probability, when known, may be safely made the basis of a judicial decision. The arguments of M. Quetelet, as well as those of MM. Laplace, Condorcet, Lacroix, and others, who have treated this part of the subject, are mathematical reasonings upon the probabilities of the truth of evidence, when the degree of credibility of each witness is supposed to be exactly known; and not attempts to find by experiment what the credibility is in any particular case. In geometry, it is absurd to expect that which is called a circle can be drawn by human hands; but not to say that, the circle being drawn, any tangent is at right angles to the radius. So in the present subject, it would be useless to pretend to assign the fraction which expresses the credibility of any one witness. Yet if we consider, that, even if these credibilities were actually given in numbers, the difficulty of forming a judgement would not be nearly all removed—that the mind is, as has been already noticed, most apt to deceive itself in the estimation of results, even in cases so simple as that of a lottery of black and white balls: we shall not be inclined to reject that branch of the subject, which teaches us how to use exact data, when they shall have been obtained, because that has not yet been done. In speaking of the caution with which the testimony of a single witness should be received, M. Quetelet cites the following, as of his own knowledge:-

‘Quelques jours après la bataille de Waterloo, un journal de pays annonça qu’un personnage auguste, ayant été blessé, pris par les ennemis et sauvé ensuite de leurs mains, jeta ses décorations à ses libérateurs en s’écriant: Mes amis, tous, vous les avez mérit’ees! Ce fait fut répété et a été cité depuis dans plusieurs ouvrages comme un des faits historiques les mieux établis. Nos descendants se garderont bien de douter de son authenticité puisqu’il a été écrit et répété sous nos yeux. Cependant nous savons vu l’auteur de ce écrit, innocemment imaginé, s’effrayer de la confiance avec laquelle il avait été reçu et ses arguments qu’on peut en déduire pour la vérité des faits historiques.’

In conclusion, we again recommend both the subject and the book to the notice of those who are engaged in teaching. If simpler parts might furnish useful and interesting exercises in arithmetic, and might relieve the monotony of the commercial questions with which our books on that subject are filled. It would at the same time help to teach that caution and self-suspicion which is, to say the least, not the predominant result of our methods of education.