Hypothesis Testing

Proportions Step By Step, with Examples
Step By Step

If $H_A$ involves $\neq$

- Solve $SD(\hat{\rho})$; find the $z$-score; the $\rho$-value is twice the small area of the tail*.

- If the $\rho$-value is less than significance (that is, $\rho < \alpha$), then Reject $H_0$; otherwise, do not reject $H_0$.

*See “Clarification” Slide
Step By Step

If $H_A$ involves $<$ or $>$

- Solve $SD(\hat{\rho})$. Sketch the Distribution of $\hat{\rho}$ (Normal w/ mean = $\rho_{H0}$, sd = $SD(\hat{\rho})$). If $H_A$ involves $\rho<$, draw a left-facing arrow over the distribution, if $\rho>$, draw a right-facing arrow.

- Mark the location of the observed $\hat{\rho}$. If the arrow is pushing the curve towards $\hat{\rho}$, then find the $z$-score. The $p$-value is the small area of the tail*.

- If the $p$-value is less than the Significance (that is, $p < \alpha$), then Reject $H0$; otherwise, do Not Reject $H0$.

*See “Clarification” Slide
Step By Step

If HA involves < or >, cont’d

- If the arrow is pushing the curve away from the observed $\hat{p}$, then HA is not a viable alternative, and we must Fail to Reject H0.

- If for some reason we want the $p$-value: solve SD ($\hat{p}$); find the $z$-score; the $p$-value is one minus the small area of the tail*. 

*See next Slide
“Small area of the tail” refers to the area away from the center of the bell curve. This $p$-value must be less than 0.50. That is, if the $z$-score is positive, flip the sign to make it negative and use the table on page A-98.
One complication with testing $H_0$ against $H_A$ is that $H_A$ may be entirely unacceptable, that is, it may ask us to move away from reasonable values of $p_{\text{hat}}$. When this happens, we cannot Reject $H_0$.

The preceding method deals with this problem and also the problem of figuring out which side of the bell curve from under which to measure area for the $p$-value -- you always use the the small area.
Throughout all the following examples, we must always first test our assumptions:

- Independence
- Randomness
- We expect at least 10 Successes and 10 Failures
- We are sampling with replacement, or we are not drawing more than one-tenth of the whole population.
Examples

In the following examples, the bottom graph will illustrate the previously described process of plotting the distribution of \( \hat{p} \) and using an arrow to show which way the alternate Hypothesis seeks to move the distribution. The purple line is the significance cut-off, the black line marks the observed \( \hat{p} \).

The top graph illustrates the Sampling Distribution with a 95% Confidence Interval (orange lines).
Necessary Formulas

\[ sd(\hat{p}) = \sqrt{\frac{p_{H0} \cdot (1 - p_{H0})}{n}} \]

\[ z = \frac{\hat{p} - p_{H0}}{sd(\hat{p})} \]
Example I

\[
\begin{align*}
p_{\text{hat}} &= 0.569, \\
p_{\text{H0}} &= 0.517, \\
n &= 550, \\
\text{se}(p_{\text{hat}}) &= 0.021116, \\
\text{sd}(p_{\text{hat}}) &= 0.021308, \\
\text{CI} &= (0.52761, 0.61039), \\
p_{\text{val}} &= 0.007335
\end{align*}
\]
**Example I**

The Sampling Distribution (The Distribution of $p$ Given The Observation)

$p_{H0}$ is outside the Confidence Interval, so the Null Hypothesis is surprising.

The Distribution of $p_{\hat{}}$ Under The Null Hypothesis

Arrow is pushing curve towards $p_{\hat{}}$

The Alt Hypothesis may be better that the Null

The observed value $p_{\hat{}}$ is outside the cutoff, so we REJECT the Null Hypothesis.
Example II

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\[ p_{\text{hat}} = 0.5496, \]
\[ p_{H0} = 0.5, \]
\[ n = 550, \]
\[ \text{se}(p_{\text{hat}}) = 0.021215, \]
\[ \text{sd}(p_{\text{hat}}) = 0.02132, \]
\[ \text{CI} = (0.50802, 0.59118), \]
\[ p_{\text{val}} = 0.0099974 \]
Example II

The Sampling Distribution (The Distribution of p Given The Observation)

The Distribution of p_hat Under The Null Hypothesis

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p_H0 is outside the Confidence Interval, so the Null Hypothesis is surprising.

Arrow is pushing curve towards p_hat. The Alt Hypothesis may be better than the Null.

The observed value p_hat is outside the cutoff, so we REJECT the Null Hypothesis.
Example III

\( H_A: p > 0.4 \)

\[
\begin{align*}
p_{\text{hat}} &= 0.33, \\
p_{\text{H0}} &= 0.4, \\
n &= 50, \\
\text{se}(p_{\text{hat}}) &= 0.066498, \\
\text{sd}(p_{\text{hat}}) &= 0.069282, \\
\text{CI} &= (0.19967, 0.46033), \\
p_{\text{val}} &= 0.84384
\end{align*}
\]
Example III

The Sampling Distribution (The Distribution of \(p\) Given The Observation)

\[ p_{H0} \text{ is inside the Confidence Interval, so the Null Hypothesis is not surprising} \]

The Distribution of \(p_{\text{hat}}\) Under The Null Hypothesis

\[ \text{Arrow is pushing curve away from } p_{\text{hat}} \]
\[ \text{The Alt Hypothesis is worse than the Null} \]
\[ \text{we cannot reject } H0 \]

\[ \text{The observed value} \]
\[ p_{\text{hat}} \text{ is not outside the cutoff, so we CANNOT REJECT the Null Hypothesis} \]