

Quick Lookup for 2-Factor, 2-Level Probabilities

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We have in mind some Sample Space. To each member of this Space 2 Categorical Variables, or “Factors”, apply. Each Factor has two and only two states (or “Levels”).

For example, our Sample Space may be the U.S. population. One variable might be gender, the other, say, employment status. The two levels of gender are then male and female, the two levels of employment status are employed and unemployed. For computation, we name these levels. We might have \mathbf{A} denote maleness, so then NOT \mathbf{A} , or \mathbf{A}^C , denotes femaleness. Likewise, we could have \mathbf{B} denote employed, and \mathbf{B}^C denote unemployed. The term $P(\mathbf{A})$ is the probability that a case in our sample space belongs to the level \mathbf{A} — that is, in our example, $P(\mathbf{A})$ is the probability that a member of the U.S. population is male.

Here are essential equalities:

$$P(\mathbf{A}) = \frac{P(\mathbf{A} \cap \mathbf{B})}{P(\mathbf{B} | \mathbf{A})} \quad (1)$$

$$P(\mathbf{A}) = P(\mathbf{A} \cap \mathbf{B}) + P(\mathbf{A} \cap \mathbf{B}^C) \quad (2)$$

$$P(\mathbf{A}) = P(\mathbf{A} | \mathbf{B}) \cdot P(\mathbf{B}) + P(\mathbf{A} | \mathbf{B}^C) \cdot P(\mathbf{B}^C) \quad (3)$$

$$P(\mathbf{A}) = 1 - P(\mathbf{A}^C) \quad (4)$$

$$P(\mathbf{A} | \mathbf{B}) = \frac{P(\mathbf{A} \cap \mathbf{B})}{P(\mathbf{B})} \quad (5)$$

$$P(\mathbf{B} | \mathbf{A}) = \frac{P(\mathbf{A} \cap \mathbf{B})}{P(\mathbf{A})} \quad (6)$$

$$P(\mathbf{B} | \mathbf{A}) = \frac{P(\mathbf{A} | \mathbf{B}) \cdot P(\mathbf{B})}{P(\mathbf{A})} \quad (7)$$

$$P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{B} | \mathbf{A}) \cdot P(\mathbf{A}) \quad (8)$$

$$P(\mathbf{B} \cup \mathbf{A}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A} \cap \mathbf{B}) \quad (9)$$

Notice that if we combine (3) and (7) we get Bayes’s Rule.