Ex. 8 / page 54

$P(A)= .3$, $P(B)=.5$

$A \cap B = \varnothing$

a) either A or B occurs : $P(A \cup B)=P(A)+P(B)=.8$
b) A occurs but B does not : $P(A \cap B')= P(A)=.3$ (because if $AB=\varnothing$ then A included in B' ,hence $AB'=A$)
c) Both A and B occur : $P(A \cap B)=P(\varnothing)=0$

Ex. 11 / page 54

Let A= American males who smoke cigarettes and B= American males who smoke cigars
Then we know that $P(A)=.28$, $P(B)=.07$, $P(A \cap B)=.05$

a) $P(A'\cap B')= P((A\cup B)')=1-P(A\cup B)=1-(P(A)+P(B)-P(A\cap B))=1-(.28+.07-.05)=.7$
b) $P(A'\cap B) = P(B)-P(A\cap B)=.02$

Ex. 18 / page 55

Let A be the event that the first card is ace and B the event that the second card is either a ten ,a jack ,a queen ,or a king.

Then $P(A\cap B)= P(A) P(B|A)=(4/52)*(4*4/51)=(4*16)/(52*51)$ (*)
But we can also have a blackjack if the second card is an ace and the first one is a ten, a jack , a queen ,or a king, so (*) has to be multiplied by two .
Then our probability is
$(2*4*16)/(52*51)$

Ex. 35 / page 55

The probability that at least one psychologist is chosen is 1 minus the probability that no psychologist is chosen. Let us compute this last probability . We would have to choose 3 people out of the 30 psychiatrists and this is done in C(30,3) {30 choose 3 } ways. The total number of possibilities to choose 3 people from the total of 54 is C(54,3) .So, the answer for this problem is

1- {30 choose 3}/{54 choose 3}

Ex.39 / page 57

They would have to choose 3 hotels from the total of 5 and this can be done in {5 choose 3} ways .Having chosen 3 hotels ,they can go to those hotels in 3! ways ,by permuting the hotels among them .So, the number of favorable choices is 3!*{5 choose 3}= 3*4*5
For the total number of possibilities we see that each of them can go the 5 hotels ,so all three could check in into 5*5*5 different ways (including for instance the case when all three of them go into the same hotel and so on) .
Hence , the probability is equal to 3*4*5/(5*5*5)=12/25

The assumptions one has to make are the ordinary ones :

1) Their choices of the hotels are independent
2) Each hotel has an equal choice of being selected by the 3 people (so tastes , personal preferences are neglected)

Ex. 46 / page 58

If n >= 13 then it is obvious that at least two of them will have to have their birthdays in the same month .So in this case the probability is one which is of course greater than ½ .
Let us consider now the case when n<13

It is easier to look at the complementary event which is the event that all 12 people have their birthdays in different months. Then, the probability of this event will be n!*{12 choose n} / 12^n (because first we choose the n months among those 12 and then we assign those months to the n people in n! ways and 12^n is the total number of possibilities (each people has 12 possibilities for their birthday ,corresponding to the 12 months of the year) .
So, the condition which has to be imposed on n is n!*{12 choose n}/12^n <= ½ (*)

Now , we have to notice that this probability is decreasing when n increases ,because the more people we have the less is the chance that all we'll have birthdays in different months of the year . So , if for a certain n (*) is fulfilled then for any number of people greater than that n, (*) will also be fulfilled .
For n=1 we get 12/12=1 >1/2

n=2 : {12 choose 2}*2/12^2 = 11/12 > ½

n=3 : {12 choose 3}*3!/12^3 = 110/144 >1/2

n=4 : { 12 choose 4}*4!/12^4 =990/1728=.57>1/2

n=5 : { 12 choose 5}*5!/12^5=7920/20736=.38 <1/2

Hence, for $n >= 5$ the probability will always be less than ½ .

Theoretical exercises

Ex. 5 / page 59

We construct the sets $F_i = E_i \setminus (E_1 \cup E_2 \cup \dots \cup E_{(i-1)})$ .It is easy to prove now that they are mutually disjoint and that $\cup F_i = \cup E_i$

Ex. 11 / page 60

Let us prove first the Bonferroni's inequality :

It is obvious that $P(A \cup B) <= 1$ (as any probability is) .But $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

So $P(A) + P(B) - P(A \cap B) <= 1$ which can be rewritten under the form

$P(A \cap B) >= P(A) + P(B) - 1$ which is what we had to prove.

Now we can apply this to E and F and we get :

$P(E \cap F) >= .9 + .8 - 1 = .7$ q.e.d