Chapter 3

13. In a certain community, 36 percent of the families own a dog, and 22 percent of the families that own a dog also own a cat. In addition, 30 percent of the families own a cat. What is
(a) the probability that a randomly selected family owns both a dog and a cat;
(b) the conditional probability that a randomly selected family owns a dog given that it owns a cat?

38. Three cooks, A, B, and C, bake a special kind of cake, and with respective probabilities .02, .03, and .05 it fails to rise. In the restaurant where they work, A bakes 50 percent of these cakes, B 30 percent, and C 20 percent. What proportion of "failures" is caused by A?

Chapter 4

20. A gambling book recommends the following "winning strategy" for the game of roulette. It recommends that a gambler bet \$1 on red. If red appears (which has probability $\frac{18}{38}$), then the gambler should take her \$1 profit and quit. If the gambler loses this bet (which has probability $\frac{20}{38}$ of occurring), she should make additional \$1 bets on red on each of the next two spins of the roulette wheel and then quit. Let X denote the gambler's winnings when she quits.
(a) Find $P\{X > 0\}$.
(b) Are you convinced that the strategy is indeed a "winning" strategy? Explain your answer!
(c) Find $E[X]$.

29. There are two possible causes for a breakdown of a machine. To check the first possibility would cost C_1 dollars, and, if that were the cause of the breakdown, the trouble could be repaired at a cost of R_1 dollars. Similarly, there are costs C_2 and R_2 associated with the second possibility. Let p and $1 - p$ denote, respectively, the probabilities that the breakdown is caused by the first and second possibilities. Under what conditions on p, C_i, R_i, $i = 1, 2$, should we check the first possible cause of breakdown and then the second, as opposed to reversing the checking order, so as to minimize the expected cost involved in returning the machine to working order?

NOTE: If the first check is negative, we must still check the other possibility.

38. If $E[X] = 1$ and $\text{Var}(X) = 5$, find
(a) $E[(2 + X)^2]$; **(b)** $\text{Var}(4 + 3X)$.

39. A ball is drawn from an urn containing 3 white and 3 black balls. After the ball is drawn, it is then replaced and another ball is drawn. This goes on indefinitely. What is the probability that of the first 4 balls drawn, exactly 2 are white?

40. On a multiple-choice exam with 3 possible answers for each of the 5 questions, what is the probability that a student would get 4 or more correct answers just by guessing?

41. A man claims to have extrasensory perception. As a test, a fair coin is flipped 10 times, and the man is asked to predict the outcome in advance. He gets 7 out of 10 correct. What is the probability that he would have done at least this well if he had no ESP?

42. Suppose that when in flight, airplane engines will fail with probability $1 - p$ independently from engine to engine. If an airplane needs a majority of its engines operative to make a successful flight, for what values of p is a 5-engine plane preferable to a 3-engine plane?

43. A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability .2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority" decoding, what is the probability that the message will be wrong when decoded? What independence assumptions are you making?

57. Suppose that the number of accidents occurring on a highway each day is a Poisson random variable with parameter $\lambda = 3$.
(a) Find the probability that 3 or more accidents occur today.
(b) Repeat part (a) under the assumption that at least 1 accident occurs today.

Theoretical exercises (Chapter 4: 19)

19. If X is a Poisson random variable with parameter λ, show that

$$E[X^n] = \lambda E[(X + 1)^{n-1}]$$

Now use this result to compute $E[X^3]$.