STATISTICS: 475.101/102/108

Assignment 3, Semester 1, 2000
Due: 4pm Thursday, 13th April

Instructions for handing in: PLEASE

1. Use (standard) **A4** sized paper.
2. Number each page in the top centre and **print** your name legibly at the **top right hand corner** of each page with the surname or family name underlined.
3. Attach a white *Department of Statistics Assignment Cover Sheet* to the front of the assignment. Staple or clip all pages together in the extreme top left hand corner.
4. Fold the paper length-wise so that the printed side of the cover sheet faces out.
5. **Print** your name, paper number and assignment number on the outside of the cover sheet. **Print** your ID number on the inside of the cover sheet.

Notes:

- Statistics is about summarising, analysing and communicating information. Communication is an important part of statistics. For this reason you will be expected to write answers which clearly communicate your thoughts. **The mark you receive will be based on your written English** as well as your statistical/technical work.
- Assignment 3 will be marked out of 60 marks, 54 marks for questions 1-4 as shown below and 6 marks for communication, style and presentation. Please refer to Section C, Questions 10 to 13, in your Lecture Workbook for examples of how to set out your assignment answers. Your final mark will be converted to a mark out of 10 which will be recorded towards your course work.

475.10x Statistics: Term Test
6.30-7.30pm Friday 14th April

The test will be comprised of **26** multiple choice questions covering chapters 1 - 7.

Test rooms for City Campus:

Paper	Surname	Room	Paper	Surname	Room
475.101/102/108	A - B	MLT2	475.101/102/108	Lo - O	Lib B10
475.101/102/108	C - Ga	HSB1	475.101/102/108	P - Sn	Lib B15
475.101/102/108	Ge - Kr	LLT	475.101/102/108	So - Z	Lib B28
475.101/102/108	Ku - Li	ULT			

You should ensure that you can find your test room well before you have to.

If you have a problem with sitting the test at this time then you must see either David Smith or Christine Miller, in Room 226 of the Maths/Physics building, before the 11th April.

Question 1. [12 marks]

(a) The following table of probabilities was obtained from Excel.

$X \sim$ Normal(μ=23, σ=2)

x	pr($X \le x$)	x	pr($X \le x$)	x	pr($X \le x$)
15	0.0000	20	0.0668	25	0.8413
16	0.0002	21	0.1587	26	0.9332
17	0.0013	22	0.3085	27	0.9772
18	0.0062	23	0.5000	28	0.9938
19	0.0228	24	0.6915	29	0.9987

Use the above table to find the following when $X \sim$ Normal($\mu = 23$, $\sigma = 2$):

(i) pr($X \le 19$) **(ii)** pr($X < 19$)

(iii) pr($X > 21$) **(iv)** pr($24 \le X \le 27$)

(b) The following table of probabilities was obtained from MINITAB:

Normal with mean = 8.60000 and standard deviation = 1.28000

P(X <= x)	x
0.1000	6.9596
0.2500	7.7367
0.7500	9.4633
0.9000	10.2404

The number of litres of soft serve ice cream sold by an ice cream van driver in an afternoon is found to be Normally distributed with a mean of 8.6 litres and a standard deviation of 1.28 litres.

(i) What is the least amount of soft serve ice cream that is needed so that the driver can satisfy demand on 90% of afternoons?

(ii) What is the interquartile range for the ice cream sales.

(c) Use either Excel, MINITAB or a graphics calculator to solve the following problems where $X \sim$ Normal($\mu = 5.1$, $\sigma = 0.87$):

(i) What is the probability that X is greater than 6?

(ii) What is the probability that X is between 3.7 and 5.6?

(iii) What value of x gives pr($X \le x$) = 0.6?

(d) X has a mean of -3 and a standard deviation of 5 and W has a mean of 5 and a standard deviation of 3. Let X and W be independent random variables and let $Y = 3X - 3W$.

(i) What are the mean and standard deviation of Y?

(ii) What can we say about the shape of the distribution of Y?

NOTES:

- For **question 1(c) and questions 3 and 4**, when you need to use the Normal distribution for calculations, you will need to use either Excel, MINITAB or a graphics calculator. You will probably need to use the computer in several short sessions. Make sure you are prepared before you get to use the computer.

- Do not hand in any computer output. Simply use the computer package (or graphics calculator) to find the solutions. Report any probabilities to 4 decimal places.

Question 2. [4 marks]

A mother and daughter were comparing their School Certificate marks for mathematics. The mother got 63% and the daughter got 72%. The daughter was pleased that she had beaten her mother.

In the year when the mother sat the exam, the mean mark was approximately Normally distributed with a mean of 50.4 and a standard deviation of 13.5.

In the year when the daughter sat the exam, the mean mark was approximately Normally distributed with a mean of 58.0 and a standard deviation of 15.3.

(a) Comment on how the mother's and daughter's marks compare.

(b) In the year the daughter sat School Certificate, 86% of 5th form high school students sat mathematics while in the year the mother sat School Certificate, only 41% of 5th form high school students sat mathematics. In one or 2 sentences, discuss how this affects your answer for (a).

Question 3. [19 marks]

There are five 3-storeyed buildings at the Tamaki Campus and a distinctive feature of them are their external elevators. The certificates in the elevators state that they are licensed to carry at most 11 people or a weight not exceeding 800 kg.

Assume that the weight of adult males and females are approximately Normally distributed with means of 76kg and 68kg and standard deviations of 10 kg and 9kg, respectively.

(a) The following question was posed to a group of students: *What are the expected value and standard deviation for the total weight of nine randomly selected males?* One of the students answered as follows:

Let M be the weight of a single male. $M \sim \text{Normal}(\mu = 76 \text{ kg}, \sigma = 10 \text{ kg})$
Let T be the total weight of nine males.
$T = 9 \times M$. $E[T] = 9 \times E[M] = 9 \times 76 = 684$. $sd[T] = 9 \times sd[M] = 9 \times 10 = 90$.

What incorrect assumption underlies this working? Correct the calculations.

(b) **(i)** On what proportion of these occasions, when a random group of 9 males enters the elevator, would the elevator be overloaded with respect to weight?
(ii) Repeat (i) for groups of 10 males, 11 males and 12 males.
(iii) What is the maximum number of males in a group such that there is less than 5% chance of the of the elevator being overloaded?
(iv) What is the maximum number of males in a group such that there is less than 1% chance of the of the elevator being overloaded?

(c) Repeat (b) for females instead of males. Note: for (i) and (ii), simply report a table of probabilities – you don't need to show working.

(d) Based ONLY on the calculations in (b) and (c), comment on the elevator certificate.

(e) Consider one group of 6 adult males and another group of 7 adult females. What is the probability that the total weight for the group of males is more than the total weight of the group of females?

Question 4. [19 marks]

The manager of a importing company purchased a new machine for packaging ground coffee. The specifications of the machine claim that the amount of coffee put in each package will be Normally distributed with an average amount of coffee as specified and a standard deviation of 2.3 grams. The machine is set to fill packets with 305 grams of coffee.

The manager requires the machine to produce packets containing coffee weighing within the range 300-310 grams for at least 95% of the packets.

(a) Assuming that the specifications for the machine are accurate, calculate the probability that a packet of coffee contains the desired amount of coffee. Will the managers requirements be met?

A sample of 30 packets of coffee was filled and weighed. The resulting weights were as follows:

304.1, 306.9, 306.6, 301.6, 305.4, 302.6, 305.4, 305.1, 307.4, 302.4,
304.2, 306.5, 303.2, 302.7, 303.0, 301.1, 309.6, 303.9, 300.1, 303.6,
307.7, 301.7, 302.8, 306.6, 304.3, 305.6, 306.1, 308.8, 304.1, 304.5

sample size = 30, sample mean = 304.6 grams, sample standard deviation = 2.305 grams.

(b) Create a stem-and-leaf plot of the data.

(c) **(i)** Does the sample of weights appear to have an exactly Normal distribution? Briefly justify your answer.
(ii) Are there any features of the stem-and-leaf plot that suggest major departures from the Normal distribution? Briefly justify your answer.
(iii) Based on (i) and (ii), is it plausible that the sample data could have come from a Normal distribution?

(d) Let $\overline{X}$ be the mean weight from a sample of 30 packets of coffee. Assuming that the specifications for the machine are accurate, give the distribution and parameters for $\overline{X}$. Was the central limit theorem needed to answer this question? Briefly justify your answer.

(e) Using the distribution from (d), what is the probability of getting a sample of 30 weights with a mean of at most 304.6 grams? State whether it would it be very unusual for this to occur.

(f) Let $\hat{P}$ be the proportion of packets of coffee outside the 300 – 310 gram weight range from a sample of 500 packets of coffee. Assuming that the specifications for the machine are accurate, give the distribution and parameters for $\hat{P}$.

A sample of 500 packets of coffee are produced. Of these, 11 were found to be outside the 300 – 310 gram weight range.

(g) Using the distribution from (f), what is the probability of getting a sample of 500 packets with at least 11 of them being outside the 300 – 310 gram weight range? State whether it would be very unusual for this to occur.

(h) Write a **brief** report (no more than 4 sentences) discussing whether or not the specifications of the machine appear accurate.