Solution to homework 3

Problem 3_1

i) Pr(X<=19) = 0.0228
ii) Pr(X<19) = 0.0228
iii) Pr(X>21) = 1-0.1587 = 0.8413
iv) Pr(24<=X<=27) = 0.9772-0.6915 = 0.2857

Problem 3_2

i) The amount needed is 10.2404 liters
ii) The IQR is 1.7266 liters
iii) Pr(X>6) = Pr(Z>(6-8.6)/1.28) = Pr(Z>-1.4444) = 0.9251

Problem 3_3

i) E(Y) = E(3X-3W) = 3E(X)-3E(W) = 3*(-3) – 3*5 = -24
SD(Y) = $(9*SD(X)^2 + 9*SD(W)^2)^{1/2}$ = $(9*25+9*9)^{1/2}$ = 17.493
ii) Random variable Y follows a normal distribution, since it's a linear combination of two normal random variables.

Problem 3_4

Let X denote the number of bits in a message that are corrupted in during transmission In this situation, X follows Binomial distribution (n, p), where n = 10^5, p = $2*10^{-5}$. In Poisson approximation, X ~ Poisson (np = 2), therefore, $P(X = x) = \frac{2^x}{x!}e^{-2}$

The probability that this message is seriously degraded is Pr (X> $2*10^{-5}$* 0.001%)

$$
\begin{aligned}
&= P(X > 2) \\
&= 1 - P(X = 0) - P(X = 1) \\
&= 1 - \frac{2^0}{0!}e^{-2} - \frac{2^1}{1!}e^{-2} \\
&= 0.594
\end{aligned}
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