UCLA STAT 10 Statistical Reasoning - Midterm Review Solutions
Observational Studies, Designed Experiments & Surveys

1. (4)

2. (3)

3. **(i)** The treatment being compared is:

Study 1: the number of storeys from which the cat fell.

Study 2: the gender of the student.

Study 3: the style of the commercial.

(ii) Study 1: An observational study. There is no allocation (by the researcher) of cats to the number of storeys of the fall. Results are simply observed for cases that happen.

Study 2: An observational study. There is no allocation (by the researcher) of subjects (the students) to the groups (male or female).

Study 3: An experiment. The researcher allocates which commercial is to be watched by each subject (shopper).

(iii) It is not possible to do an experiment for study 1 due to ethical and moral considerations. To do an experiment a sample of cats would have to be allocated a height and then thrown out of a window at that height.

It is not possible to do an experiment for study 2 as the researcher cannot allocate a gender to a student.

4. (5)

5. (3)

6. (4)

7. Self-selection, selection bias.

8. **(i)** False. A sample of 500 is large enough to give a useful indicative result.

(ii) True. Homeless people cannot be contacted by telephone.

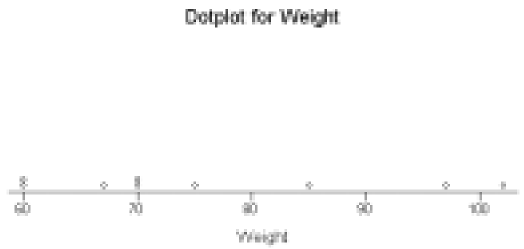
(iii) True. Telephone polls will usually have some non-response bias. There is no indication of how vigorously non-respondents were followed up.

(iv) False. Control groups are not necessary in polls or surveys. Control groups are often used when comparisons want to be made (ie, in observational studies and experiments).

9. (2)

Graphs, Numerical Summaries, Histograms

1. **(a)**



(b) The centre of these data is about 75kg.
The data is skewed to the right (positively skewed).
There are gaps at weights at around 80kg and 90kg.

2. **(a)** Units: 9 | 5 = 950Å

8	4
8	
9	0 3 4
9	5 6 7 8 9 9
10	0 0 0 1 1 3 3 3 4 4
10	5 5 5 5 5 7 7
11	0 0 2

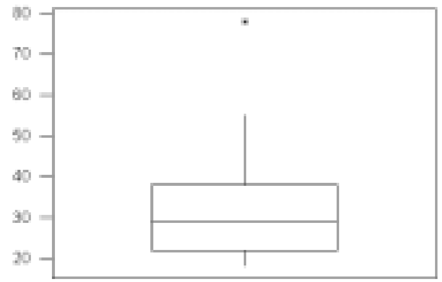
(b) (5) **(c)** (5) **(d)** (2)

3. (3)

4. (2)

5. **(1)** Agree
(2) Agree
(3) Disagree
(4) Disagree

6.



7. **(1)** Disagree, the skew is only moderate.
(2) Disagree, the interquartile range is 16.
(3) Agree.
(4) Agree.
(5) Disagree.

8. (1)

9. (2)

10. (3)

11. (3)

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Exploratory Tools for Relationships

Section A: Types of Variables

1. **(a)** **Quantitative** variables are **measurements** and counts.

   (b) **Qualitative** variables describe **group membership**.

2. **(a)** Variables with **few** *repeated values* are treated as **continuous**.

   (b) Variables with **many** *repeated values* are treated as **discrete**.

3. **(a)** Variables **with order** are called **ordinal**.

   (b) Variables **without order** are called **categorical**.

4. **(a)** To explore the relationship between two **quantitative** variables we use a **scatter plot**.

   (b) To explore relationships between a **qualitative** variable and a **quantitative** variable we use **dot** plots, **stem-and-leaf** plots and **box** plots.

   (c) To explore the relationship between two **qualitative** variables we use a **two-way table** of **counts**.

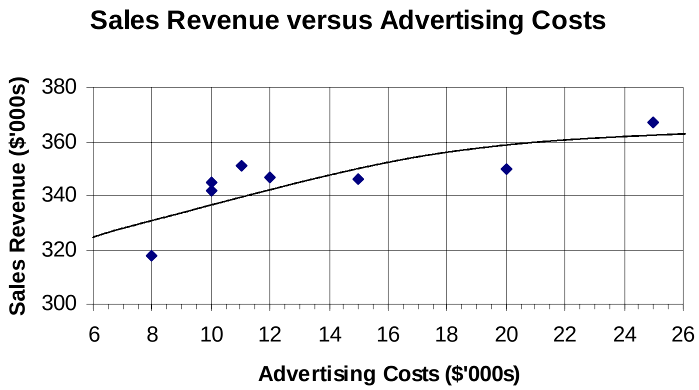
Section B: Two Variables

1. (1)

2. (3)

3. (5)

4.

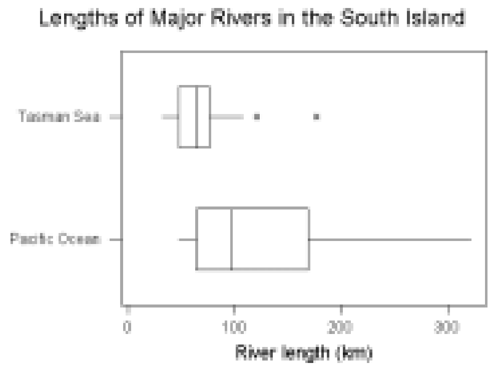


Interpretation:
As advertising costs increase, sales revenue also increases.
The relationship is not linear – the increase in sales revenue decreases as advertising costs increase.
There are no outliers in this data.
The amount of scatter about the trend curve is small.

5. **(a)** **Pacific Ocean rivers:**
Med = 97
Q_1 = 64
Q_3 = 169

Tasman Sea rivers:
Med = 64
Q_1 = 48
Q_3 = 76

(b)



Note: For the rivers flowing into the Pacific Ocean:
IQR = 105, 1.5 x IQR = 157.5, Q_3 + 1.5 x IQR = 326.5, Q_1 – 1.5 x IQR = -93.5
There are no outside values, the whiskers end at 48 (lower) and 322 (upper).
For the rivers flowing into the Tasman Sea:
IQR = 28, 1.5 x IQR = 42, Q_3 + 1.5 x IQR = 118, Q_1 – 1.5 x IQR = 6
177 and 121 are outside values, the whiskers end at 32 (lower) and 108 (upper).

(c) On average, the rivers flowing into the Pacific Ocean are longer.

The lengths of the rivers flowing into the Pacific Ocean have a larger spread than the lengths of the rivers flowing into the Tasman Sea.

The lengths of the rivers flowing into the Pacific Ocean are skewed to the right (positively skewed).

The Grey River and Buller River are outliers amongst the rivers flowing into the Tasman Sea.

Chapter 4 – Probabilities and Proportions

1. **(a)**

	Females	Males	**Total**
528.181	0.3003	0.2949	0.5952
528.188	0.1924	0.2123	0.4048
Total	0.4928	0.5072	1

(b) **(i)** $\frac{593}{2011} = 0.2949$ **(ii)** $\frac{991}{2011} = 0.4928$ **(iii)** $\frac{387}{2011} = 0.1924$

(c) $\frac{427}{814} = 0.5246$

(d) $\frac{427}{1020} = 0.4186$

2. **(a)**

	40 or under	Over 40	**Total**
Wearing a seat belt	0.484	**0.369**	0.853
Not wearing seat belt	**0.066**	0.081	**0.147**
Total	**0.55**	**0.45**	1

(b) 0.147

(c) $\frac{0.081}{0.147} = 0.551$

(d) 0.55

3. **(a)**

	Under 40	40 or over	Total
Mild cases	16	20	36
Serious cases	15	35	50
Total	31	55	86

(b) **(i)** $\frac{16}{86} = 0.1860$ **(ii)** $\frac{20+35+15}{86} = 0.8140$ **(iii)** $\frac{35}{86} = 0.4070$

(c) $\frac{15}{50} = 0.3$

(d) $\frac{20}{55} = 0.3636$

4. **(a)**

	High-risk	Low-risk	Total
In default	40% of 0.05 = 0.02	0.03	0.05
Not in default	0.2185	77% of 0.95 = 0.7315	0.95
Total	0.2385	0.7615	1

(b) 23.85%

(c) $\frac{0.02}{0.2385} = 0.0839$

5. **(a)**

	Female	Male	Total
Business degree	27% of 0.175 = 0.04725	0.12775	0.175
Other degree	0.42281	48.75% of 0.825 = 0.40219	0.825
Total	0.47006	0.52994	1

(b) 0.52994

(c) 0.12775

(d) $\frac{0.04725}{0.47006} = 0.1005$

6. (2)

Chapter 5 – Discrete Random Variables

Section A: Discrete Random Variables

1. **(a)**

x	**0**	**1**	**2**
pr($X = x$)	**0.25**	**0.5**	**0.25**

(b) **(i)** pr($X > 1$) = 0.25 **(ii)** pr($X \geq 1$) = 0.75 **(iii)** pr($X \leq 2$) = 1

2. **(a)** pr($Y > 12$) = 0.03 + 0.28 + 0.12 = 0.43
(b) pr($Y \leq 10$) = 0.10 + 0.07 + 0.25 = 0.42
(c) pr($Y \geq 6$) = 1 – pr($Y < 6$) = 1 – 0.10 = 0.9
(d) pr($6 \leq Y \leq 12$) = 0.07 + 0.25 + 0.15 = 0.47
(e) pr($10 \leq Y \leq 12$) = 0.25 + 0.15 = 0.4
(f) pr($13 < Y < 25$) = 0.28

Section B: Binomial Distribution

1. Let X be the number of customers who purchase at least one book.
X ~ Binomial (n = 7, p = 0.3)
pr($X \geq 2$) = 1 – pr($X \leq 1$) = 1 – 0.329 = 0.671

2. **(a)** n = 10, p = 0.05
(b) There is a fixed number of trials, 10. Each disk drive is a trial.
Each trial has 2 outcomes: Disk drive malfunctions or disk drive does not malfunction.
The disk drives are independent.
The probability that a disk drive malfunctions is constant.
(c) The first two assumptions will be satisfied.
The disk drives may not be independent. Disk drives could be made from the same batch of materials or may have the same systematic fault.
The probability of a disk drive malfunctioning will not be constant because it will depend on how a disk drive is used.
(d) **(i)** pr($X = 0$) = 0.599
(ii) pr($X = 1$) = 0.315
(iii) pr($X \geq 2$) = 1 – pr($X \leq 1$) = 1 – 0.914 = 0.086
(iv) pr($2 \leq X \leq 5$) = pr($X \leq 5$) – pr($X \leq 1$) = 1.00 – 0.914 = 0.086

Section C: Poisson Distribution

1. **(a)** The events occur at a constant average rate of λ per unit time.
(b) Occurrences are independent of one another.
(c) The probability of 2 or more occurrences in a time interval of length d tends to zero, as d tends to zero.

2. **(a)** 0.143 **(b)** 0.038 **(c)** 1 – 0.406 = 0.594 **(d)** 0.522 **(e)** 0.792 – 0.043 = 0.749

Section D: Differences Between Poisson and Binomial Distributions

1. (b) **2.** (d) **3.** (b)

Section E: Choosing an Appropriate Probability Model

1. There is a fixed number of trials, 25 boxes. Each trial has 2 outcomes, water damaged or not. But sampling is from a finite population of 1200 boxes so conditions will not be the same for each trial. However $\frac{n}{N} = \frac{25}{1200} \approx 0.02 < 0.1$ so conditions for each trial will be approximately the same.
An appropriate model is X ~ Binomial (n = 25, $p = \frac{200}{1200}$)

2. Neither.

3. There are two possible answers.
(a) There is a constant average rate for cataracts as there is nothing to suggest that births with cataracts occur in greater numbers at varying times in the year. The occurrence of a baby being born with cataracts is probably independent of other occurrences of babies being born with cataracts. The probability of two or more babies being born with cataracts in an interval of size d approaches 0 as d approaches 0, ie occurrences of babies being born with cataracts won't happen at exactly the same time.
An appropriate model is X ~ Poisson (λ = 1.5)
(b) There is a fixed number of trials (50,000 babies); each baby either has cataracts or not (2 outcomes), occurrences of cataracts are probably independent, and the probability that a baby is born with cataracts is constant (0.00003).
An appropriate model is X ~ Binomial (n = 50,000, p = 0.00003)

4. There are 3 trials (each coin is a trial). There are 2 outcomes for each trial (head or tail). Each coin is independent of the other coins. The probability of a head being tossed is 0.5 for each coin.
An appropriate model is X ~ Binomial (n = 3, p = 0.5)

5. The average rate may not be exactly constant over the 6-month period because strokes may occur more often at one time of the year.
Deaths due to strokes are likely to be independent.
Deaths by stroke are unlikely to occur at exactly the same time.
An appropriate model is X ~ Poisson (λ = 275)

6. There are 45 trials (each person who has bought a new car is a trial). There are 2 outcomes for each trial (wins a major prize or not). Each person's result is independent of the other people's result. The probability of a major prize being won is 0.1 for each person.
An appropriate model is X ~ Binomial (n = 45, p = 0.1)

7. Neither.

8. Fixed number of trials, 200 students. Each trial has 2 outcomes, student has not done any mathematics in the previous two years or they have. But sampling is from a finite population of 1400 boxes so conditions will not be the same for each trial. However $\frac{n}{N} = \frac{200}{1400} \approx 0.14 > 0.1$ so conditions for each trial will not be approximately the same.
Neither the Binomial distribution nor the Poisson distribution is appropriate.

Chapter 6 – Continuous Random Variables

Section A: Probability Density Function Quiz

1. Areas under the density curve represent probabilities. The probability that a random observation falls between a and b is equal to the area between the density curve and the x-axis from $x = a$ and $x = b$.

2. The total area under the curve equals 1.

3. The population mean μ is the point where the density curve balances.

4. No, because for a continuous random variable:

$\text{pr}(a \le X \le b) = \text{pr}(a < X \le b) = \text{pr}(a \le X < b) = \text{pr}(a < X < b)$ = area under the curve between a and b.

5. The curve is bell-shaped, symmetrical and centred at μ. The standard deviation σ governs the spread.

6. The parameters are μ and σ.

Section B: Normal Distribution

1. **(a)** $\text{pr}(X < 245) = 0.0947$

(b) $\text{pr}(255 < X < 280) = \text{pr}(X < 280) - \text{pr}(X < 255) = 0.8092 - 0.2459 = 0.5633$

(c) $\text{pr}(X > 287) = 1 - \text{pr}(X < 287) = 1 - 0.9053 = 0.0947$

2. Let X be the survival time in months of a cancer patient on this drug.

(a) **(i)** $\text{pr}(X \le 12) = 0.1163$

(ii) $\text{pr}(12 < X < 24) = \text{pr}(X < 24) - \text{pr}(X < 12) = 0.3286 - 0.1163 = 0.2123$

(iii) $\text{pr}(X > x) = 0.8$ therefore $\text{pr}(X < x) = 0.2$ and so $x = 17.6341$.

80% of the patients live beyond 17.6 months.

(iv) $\text{pr}(a < X < b) = 0.8$

$\text{pr}(X < a) = 0.1$ and so $a = 10.5932$

$\text{pr}(X < b) = 0.9$ and so $b = 51.6048$

The range of the central 80% of survival times is from 10.6 to 51.6 months.

(b) There are some doubts about the validity of the assumption that survival times are Normally distributed. Although the data is roughly symmetrical, there is a gap in the centre which could indicate bimodality of survival times. The tails seem too short for the underlying distribution to have a Normal distribution.

3. Let X be the maximum distance reached by a pilot without moving the seat.

(a) $\text{pr}(X \ge 120) = 1 - \text{pr}(X \le 120) = 1 - 0.3085 = = 0.6915$

(b) $\text{pr}(X \ge x) = 0.95$ therefore $\text{pr}(X < x) = 0.05$ and so $x = 108.5515$.

The maximum distance at which the switch should be placed is 109cm.

(c) **(i)** That this pilot's maximum reach is 1.5 standard deviations above the mean.

(ii) $x = 125 + 1.5 \times 10 = 140$cm. A z-score of 1.5 corresponds to a maximum reach of 140cm.

Section C: Combining Random Variables

1. **(a)** $\boxed{W} = \boxed{3} \times \boxed{X} + \boxed{2} \times \boxed{Y}$

(b) $\boxed{T} = \boxed{3} \times \boxed{X} + \boxed{-2} \times \boxed{Y}$

(c) $\boxed{V} = \boxed{1} \times \boxed{Y} + \boxed{-1} \times \boxed{X}$

2. (4)

3. (1)

4. (5)

5. (1)

6. **(a)** Let G_i be the charge for a randomly chosen gardening job. $G_i \sim \text{Normal}(\mu = 25, \sigma = 3)$

$X = G_1 + G_2 + G_3 + G_4 + G_5 + G_6$

$E(X) = E(G_1 + G_2 + G_3 + G_4 + G_5 + G_6) = 6 \times E(G_i) = 6 \times 25 = 150$

$\text{sd}(X) = \text{sd}(G_1 + G_2 + G_3 + G_4 + G_5 + G_6) = \sqrt{6} \times \text{sd}(G_i) = \sqrt{6} \times 3 = 7.35$

(b) **(i)** Let M_i be the charge for a randomly chosen mowing job. $M_i \sim \text{Normal}(\mu = 15, \sigma = 2)$

$Y = M_1 + M_2 + \ldots + M_{11}$

$E(Y) = E(M_1 + M_2 + \ldots + M_{11}) = 11 \times E(M_i) = 11 \times 15 = 165$

$\text{sd}(Y) = \text{sd}(M_1 + M_2 + \ldots + M_{11}) = \sqrt{11} \times \text{sd}(M_i) = \sqrt{11} \times 2 = 6.63$

(ii) $T = X + Y$

(iii) T has a Normal distribution because it is a combination of Normally distributed random variables.

(iv) $E(T) = E(X + Y) = E(X) + E(Y) = 150 + 165 = 315$

$\text{sd}(T) = \text{sd}(X + Y) = \sqrt{\text{sd}(X)^2 + \text{sd}(Y)^2} = \sqrt{7.35^2 + 6.63^2} = 9.90$

In order to calculate the standard deviation of T we had to assume that X and Y are independent random variables.

(c) Let M be the charge for a randomly chosen mowing job. $M \sim \text{Normal}(\mu = 15, \sigma = 2)$

$W = 52M$

$E(W) = 52E(M) = 52 \times 15 = 780$

$\text{sd}(W) = |52|\text{sd}(M) = 52 \times 2 = 104$

$W \sim \text{Normal}(\mu = \$780, \sigma = \$104)$

7. Let M be the morning travel time and N be the evening travel time.

(a) Want pr($M + N > 60$)

E($M + N$) = E(M) + E(N) = 31 + 35.5 = 66.5 minutes

sd($M + N$) = $\sqrt{\text{sd}(M)^2 + \text{sd}(N)^2} = \sqrt{3^2 + 3.5^2}$ = 4.60977, assuming M and N are independent.

$M + N$ ~ approx. Normal (μ = 66.5, σ = 4.60977)

pr($M + N > 60$) = 1 – pr($M + N \leq 60$) = 1 – 0.0793 = 0.9207

(b) Want pr($M > N$) = pr($M - N > 0$)

E($M - N$) = E(M) – E(N) = 31 – 35.5 = –4.5 minutes

sd($M - N$) = $\sqrt{\text{sd}(M)^2 + \text{sd}(N)^2} = \sqrt{3^2 + 3.5^2}$ = 4.60977, assuming M and N are independent.

$M - N$ ~ approx. Normal (μ = –4.5, σ = 4.60977)

pr($M - N > 0$) = 1 – pr($M - N < 0$) = 1 – 0.8355 = 0.1645

(c) Want pr($N - M > 5$)

E($N - M$) = E(N) – E(M) = 35.5 – 31 = 4.5 minutes

sd($N - M$) = $\sqrt{\text{sd}(N)^2 + \text{sd}(M)^2} = \sqrt{3.5^2 + 3^2}$ = 4.60977, assuming N and M are independent.

$N - M$ ~ approx. Normal (μ = 4.5, σ = 4.60977)

pr($N - M > 5$) = 1 – pr($N - M < 5$) = 1 – 0.5432 = 0.4568

(d) **(i)** Let $T_M = M_1 + M_2 + M_3 + M_4 + M_5$, where M_i ~ approx. Normal (μ = 31, σ = 3)

E(T_M) = E($M_1 + M_2 + M_3 + M_4 + M_5$) = 5E($M$) = 5 x 31 = 155 minutes

sd(T_M) = sd($M_1 + M_2 + M_3 + M_4 + M_5$) = $\sqrt{5} \times \text{sd}(M)$ = $\sqrt{5} \times 3$ = 6.7082 minutes, assuming independence of the morning travel times.

T_M ~ approx. Normal (μ = 155min, σ = 6.7082min)

(ii) Let $T_N = N_1 + N_2 + N_3 + N_4 + N_5$, where N_i ~ approx. Normal (μ = 35.5, σ = 3.5)

E(T_N) = E($N_1 + N_2 + N_3 + N_4 + N_5$) = 5E($N$) = 5 x 35.5 = 177.5 minutes

sd(T_N) = sd($N_1 + N_2 + N_3 + N_4 + N_5$) = $\sqrt{5} \times \text{sd}(N)$ = $\sqrt{5} \times 3.5$ = 7.8262 minutes, assuming independence of the morning travel times.

T_N ~ approx. Normal (μ = 177.5min, σ = 7.8262min)

(iii) Let $T = T_M + T_N$

E(T) = E($T_M + T_N$) = E(T_M) + E(T_N) = 155 + 177.5 = 332.5 minutes

sd(T) = sd($T_M + T_N$) = $\sqrt{\text{sd}(T_M)^2 + \text{sd}(T_N)^2}$ = $\sqrt{6.7082^2 + 7.8262^2}$ = 10.3078 minutes, assuming independence of ten travel times in the week.