Stat13 Homework 4

http://www.stat.ucla.edu/~dinov/courses_students.html

(30 points, student scores will be converted to scores out of 100)

Suggested Solutions

Problem 1

a) (4 points)

$\Pr(X \le 19) = 0.0228$

$\Pr(X < 19) = 0.0228$

$\Pr(X > 21) = 1 - 0.1587 = 0.8413$

$\Pr(24 \le X \le 27) = 0.9772 - 0.6915 = 0.2857$

b) (2 points)

Let $least$ denote the least amount needed. Then $least$ satisfies: $\Pr(X \le least) = 90\%$.

We can see from the table that the least amount needed is 10.2404

The IQR is 9.4633-7.7367=1.7266

c)

i) (1 point)

$$z = \frac{6 - 5.1}{0.87} = 1.0345$$

$\Pr(X > 6) = \Pr(Z > 1.0345) = 1 - 0.848 = 0.152$

ii) (1 point)

similarly,

$$\Pr(4.3 \le X \le 6.6) = \Pr\left(\frac{4.3 - 5.1}{0.87} \le Z \le \frac{6.6 - 5.1}{0.87}\right) = 0.9577 - 0.1789 = 0.7788$$

iii) (1 point)

Pr(Z<-0.126)=0.45

Hence $x = -0.126 \times 0.87 + 5.1 = 4.99$

d)

i) (1 point for mean, 1 points for SD)

$E(Y) = 3E(X) - 3E(W) = -24$

$$SD(Y) = \sqrt{3^2 SD^2(X) + 3^2 SD^2(W)} = \sqrt{306} = 17.49$$

ii) (2 point)

----centered much lower than either X or W

----distribution is more spread out than X or W

Problem 2

i) (4 points)

z-score for the value:

-6: $z = \frac{-6 - 3}{4} = -2.25$, hence 2.25 SD away from mean

10: $z = \frac{10 - 3}{4} = 1.75$, hence 1.75 SD away from mean

7: $z = \frac{7 - 3}{4} = 1$, hence 1 SD away from mean

-0.4: $z = \frac{-0.4-3}{4} = -0.825$, hence 0.825 SD away from mean

ii) (2 points)

$\Pr(-2 \le X \le 0) = \Pr(\frac{-2-3}{4} \le Z \le \frac{0-3}{4}) = 0.2266 - 0.1056 = 0.121$

No difference. Because $\Pr(-2 < X < 0) = \Pr(-2 \le X \le 0) - \Pr(X = -2) - \Pr(X = -0)$.

But $\Pr(X = -2) = \Pr(X = -0) = 0$ for normal distribution. Hence no difference.

Problem 3. (6 points total, 2 for each)

i) $\Pr(73 \le M \le 75) = \Pr(\frac{73-70}{3} \le Z \le \frac{75-70}{3}) = 0.9522 - 0.8413 = 0.1109$

ii) $M + F \sim N(70 + 65.5, (\sqrt{3^2 + 2.5^2})^2)$

So, $\Pr(M + F \le 140) = \Pr(Z \le \frac{140 - (70 + 65.5)}{\sqrt{3^2 + 2.5^2}}) = 0.8754$

iii) $M - F \sim N(70 - 65.5, (\sqrt{3^2 + 2.5^2})^2)$

So, $\Pr(M - F \le 0) = \Pr(Z \le \frac{0 - (70 - 65.5)}{\sqrt{3^2 + 2.5^2}}) = 0.1246$