STAT 110 A, Probability & Statistics for Engineers I
UCLA Statistics, Spring 2003

http://www.stat.ucla.edu/~dinov/courses_students.html

HOMEWORK 5

Due Date: Friday, June 06, 2003, turn in after lecture

Correct solutions to any **five** problems carry full credit. See the HW submission rules. On the front page include the following header. You may want to use the online SOCR resources to complete this assignment.

- (**HW_5_1**) [Sec. 4.4, #62] A system consists of 5 identical components connected in a series as follows:

→	1	→	2	→	3	→	4	→	5	→

As soon as one component fails, the system fails. Suppose each component has a lifetime that is exponentially distributed with $\lambda = 0.01$ and they fail independently of one another. Let A be the event that the i^{th} component lasts at least t hours.
(a) Are the A_i independent events (for i = 1, 2, 3, 4, 5)?
(b) Let X be the time at which the system fails. The event $X_t = \{X \geq t\}$ is equivalent to what event involving the A_i's?
(c) Compute $P(X_t)$. What is $F(t) = P(X_t)$?
(d) What is the density of X? What is the distribution of X?

- (**HW_5_2**) [Sec. 4.4, #48] Suppose that 10% of all steel shafts produced by a certain process are nonconforming, but can be reworked (rather than being scrapped). Consider a random sample of 200 shafts, let X denote the number among those that are nonconforming and can be reworked. What is the exact distribution of X? What (approximately) is the probability that X is:
(a) At most 30?
(b) Less than 30?
(c) Between 15 and 25?
(d) What is the 33th percentile for X?

- (**HW_5_3**) Let ***X*** & ***Y*** have a joint density function given by

$$f(x;y) = \begin{cases} (9/26)(xy+y)^2 & 0 \le x \le 2;\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

  Compute:
  (a) ***f(y | x)***, the conditional probability density function, p.d.f., of ***Y*** given ***X***.
  (b) $P(Y < 1/2 \mid X < 1/2)$.
  (c) $E(Y \mid X = x)$.

- (**HW_5_4**) [Sec. 4.1, #10 & 24] List 8 to 10 probability mass/density functions that we have discussed in class as models for various natural processes.
  (a) Give one example of a process that can be modeled by each distribution you listed.
  (b) Identify the parameters, if any, for all distributions. Discuss the shape of the distribution, if known.
  (c) If the mean and the variance of the distribution are known write them explicitly.
  (d) In your own words state the Central Limit Theorem. What is its application to this collection of distributions you have presented.

- (**HW_5_5**) [Sec. 6.2, #22] Let ***X*** denote the proportion of alotted time that s randomly selected student spends working on a certain aptitude test. Suppose the pdf of ***X*** is

$$f(x;\vartheta) = \begin{cases} (\vartheta + 1)\, x^{\vartheta} & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

  where $\vartheta > -1$. A random sample of 10 students yields the following data

0.92	0.79	0.90	0.65	0.86	0.47	0.73	0.97	0.94	0.77

  (a) Use the ***method of moments*** to obtain an estimator of ϑ and then use this to compute an actual estimate for these data.
  (b) Obtain a ***maximum-likelihood estimator*** of ϑ and use it to calculate an estimate for the given data.

- (**HW_5_6**) A cigarette manufacturer claims that his cigarettes have an average nicotine content of 1.83 milligrams. If a random sample of 8 cigarettes of this type shows a sample mean of 1.95 with sample deviation 0.22 milligrams, find 95% confidence interval for the population mean. Do you agree with the claim?

- (**HW_5_7**) Let p be the real probability of getting head when tossing a given coin. We tossed 500 times and found 260 heads. Find the 95% confidence interval for p. How many times should we toss the coin in order to be 95% confident that our estimate of p is within 0.02?

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