Stats 110B

HW1 Suggested Solutions

http://www.stat.ucla.edu/~dinov/courses_students.dir/03/Spr/Stat110B.dir/STAT110B.html

http://www.stat.ucla.edu/~dinov/courses_students.dir/03/Spr/Stat110B.dir/assignments.html

1. 3/365 = 0.008219 = relatively frequency
   Similarly 5/365 = 0.013699 = relatively frequency for interval 2
   Cumulative frequency = 0.008219 + 0.013699 = 0.02192

   The histogram is (negatively) skewed to the left. We can see that the bars are increasing in height except for the last one.

   Proportion of days that are cloudy. = 0.008219 + 0.013699 + 0.016438 + 0.021918 = 0.06027(from table)

   Proportion of clear days = 0.230137 + 0.030137 = 0.260274

2. (a) We first calculate the probability of blackout
   P(Blackout) = P(Blackout|A)*P(A) + P(Blackout|B)*P(B) + P(Blackout|C)*P(C) + P(Blackout|2 or more overload)*P(2 or more)
   = 0.01*0.6+0.02*0.2+0.03*0.15+0.05*0.05 = 0.017

   P(overload occurred at substance A alone) = P(A|Blackout)
   = P(Blackout $\cap$ A)/P(Blackout)
   = P(Blackout|A)*P(A)/P(Blackout)
   = 0.01*0.6/0.017 = 0.3529

   similarly for (b) P(B|Blackout) = P(Blackout|B)*P(B)/P(Blackout)
   = 0.02*0.2/0.017 = 0.2353

   for (c) P(C|Blackout) = P(Blackout|C)*P(C)/P(Blackout)
   = 0.03*0.15/0.017 = 0.2647

   for (d) P(2 or more simultaneously|Blackout)
   = P(Blackout|2 or more)*P(2 or more)/P(Blackout)
   = 0.05*0.05/0.017 = 0.1471

3. (a) $P(A|B) = P(A \cap B)/P(B)$
   $= (P(A)+P(B)-P(A \cup B))/P(B)$
   $= (a + b - c)/b$ for $c = P(A \cup B)$
   since $c = P(A \cup B) \leq 1$
   $P(A|B) \geq (a + b - 1)/b$

   (b) $P(A \cup B \mid C) = P((A \cup B) \cap C))/P(C) = P((A \cap C) \cup (B \cap C))/P(C)$
   $= P(A \cap C)/P(C) + P(B \cap C)/P(C) - P((A \cap C) \cap (B \cap C))/P(C)$
   $= P(A|C) + P(B|C) - P(A \cap B \cap C)/P(C)$
   $= P(A|C) + P(B|C) - P(A \cap B \mid C)$

(c) Given $P(A|B)<P(A)$

$P(A \cap B)/P(B)<P(A)$

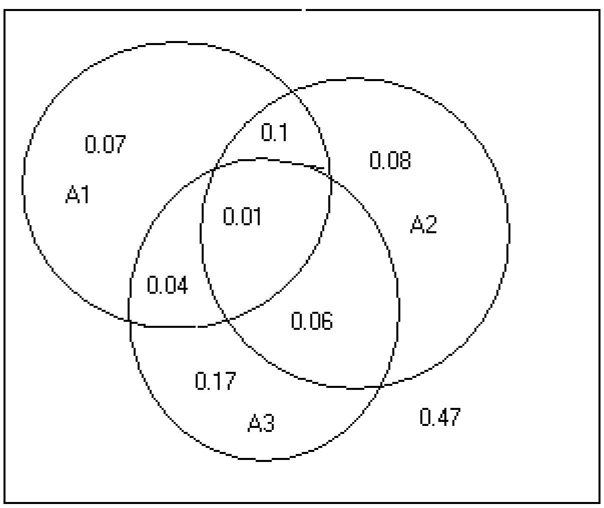
$P(A \cap B)<P(A)*P(B)$ as P(A), P(B) must be non-negative

$P(A \cap B)/P(A) < P(B)$

$P(B|A)<P(B)$

4. (a) uniform distribution. A=-5 B=5. The distribution is symmetric.
   $P(X<0) = 0.5$
   (b) $P(-2.5<x<2.5) = 0.5$
   (c) $P(-2 \le x \le 3) = P(-2<x<3) = 0.5$
   (d) $-5<k<k+4<5$
   $-5<k<1$
   $P(k<x<k+4) = 0.4$

5.



Note : not drawn to scale

(a) $A_1 \cup A_2$
   A_1 or A_2
   $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$
   $= 0.22 + 0.25 - 0.11 = 0.36$

(b) Not A_1 and not A_2
   From venn diagram, we can see that the probability is .17+.47 = 0.64

(c) A_1 or A_2 or A_3
   From diagram, the probability is the total area within the three circles
   = 0.53

(d) Not A_1 and not A_2 and not A_3
   The probability is the area of anything that is beyond the circles.
   = 1- 0.53 = 0.47

(e) Not A_1 and not A_2 and A_3

from diagram = 0.17

(f) Not A_1 and not A_2, or A_3

from diagram, probability = 0.47+0.28 = 0.75

6. (a) P(exactly one) = $(e^{-0.2} (0.2)^1)/1! = 0.1637$

(b) P(at least two) = 1-P(exactly 1)-P(exactly 0)

$= 1 - (e^{-0.2} (0.2)^1)/1! - (e^{-0.2} (0.2)^0)/0! = 0.01752$

(c) Since they are independently selected,

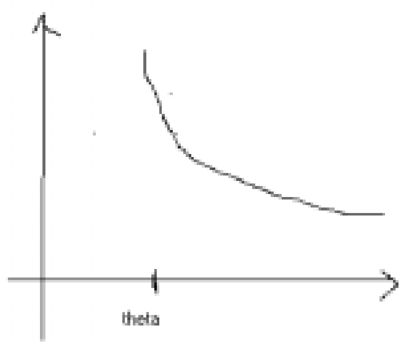
P(neither contains a missing pulse)

$= (P(\text{exactly 0 missing}))^2$

$= (0.8187)^2$

$= 0.6703$

7. (a)



(b) $\int_\theta^\infty (k\theta^k)/x^{k+1}\, dx = (-\theta^k/x^k)|_\theta^\infty = \theta^k/\theta^k = 1$

(c) $\int_\theta^b (k\theta^k)/x^{k+1}\, dx = (-\theta^k/x^k)|_\theta^b = 1 - (\theta/b)^k$ bvbvbvbvbvbvgfgfgfgfgfg

8. (a) $P(x \geq 10) = 1 - P(x \leq 10)$

$= 1 - P(z \leq (10 - 8.8)/2.8) = 1 - P(z \leq 0.42857) = 0.334$

Since inch is continuous, $P(x > 10) = P(x \geq 10) = 0.334$

(b) $P(x > 20) = 1 - P(x \leq 20) = 1 - P(z \leq (20 - 8.8)/2.8)$

$= 1 - P(z \leq 4) = 3.17 \times 10^{-5} \approx 0$

(c) $P(5 \leq x \leq 10) = P((5 - 8.8)/2.8 \leq z \leq (10 - 8.8)/2.8))$

$= P(-1.357 \leq z \leq 0.42857)$

$= P(z \leq 0.42857) - P(z \leq -1.357) = 0.5785$

(d) $P(8.8\text{-}c \leq x \leq 8.8\text{+}c) = 0.98$

$P((8.8\text{-}c\text{-}8.8)/2.8 \leq z \leq (8.8\text{+}c\text{-}8.8)/2.8) = 0.98$

$P(-c/2.8 \leq z \leq c/2.8) = 0.98$

$c = 2.33 * 2.8 = 6.524$