STAT 110 B, Probability & Statistics for Engineers II
UCLA Statistics, Spring 2003

http://www.stat.ucla.edu/~dinov/courses_students.html

HOMEWORK 3

Due Date: Friday, May 09, 2003, turn in after lecture

Correct solutions to any four problems carry full credit. See the HW submission rules. On the front page include the following header.

- (**HW_3_1**) Let X denote the velocity of a random gas molecule. According to the Maxwell-Boltzman law, the density for **X** is given by $f_X(x) = c\, x^2 e^{-\gamma x}$, $x > 0$, and $f_X(x) = 0$, $x \le 0$. Here c is a constant that depends on the physico-chemical properties of the gas involved and γ is a constant whose value depends on the mass of the molecule and its absolute temperature. The kinetic energy of the molecule, K, is given by $K = mX^2/2$, $m > 0$.

  (a) Find the density, $f_K(x)$, of K using a **direct** [start with the cdf $F_K(k) = P\{K \le k\} = \ldots = P\{X \le r\} = F_X(r)$, then differentiate the cdf to get the pdf] and **indirect** [Let **J** be the the Jacobian of the transformation from X to K, then $f_K(x) = \mathbf{J} * f_X(x)$] approaches.
  (b) For a given mass m what is the average (*expected value of the*) kinetic energy of a gas molecule, K. [Hint: When you write the expectation, do not compute the integral by hand, rather identify the integral as being the expected value for a $\Gamma(\alpha, \beta)$ distribution, for which we have discussed (see class notes) the expectation! You only need to identify α & β in terms of m & γ.]

- (**HW_3_2**) Let X & Y have a joint density function given by

$$f(x; y) = \begin{cases} (9/26)(xy+y)^2 & 0 \le x \le 2;\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

  Compute:
  (a) $f(y \mid x)$, the conditional probability density function, p.d.f., of Y given X.

(b) $P(Y < 1/2 \mid X < 1/2)$.
(c) $E(Y \mid X = x)$.

- (**HW_3_3**) List 10 probability mass/density functions that we have discussed in class as models for various natural processes.
(a) Give one example of a process that can be modeled by each distribution you listed.
(b) Identify the parameters, if any, for all distributions. Discuss the shape of the distribution, if known.
(c) If the mean and the variance of the distribution are known write them explicitly.
(d) In your own words state the Central Limit Theorem. What is its application to this collection of distributions you have presented.

- (**HW_3_4**) Give one practical and complete example of what the ***bias*** and ***precision*** of estimators are and what they are used for. Why is *sample averaging* an unbiased estimation technique for the distribution parameter we call *population mean*.

- (**HW_3_5**) Suppose the data below are a sample of midterm exam scores for one course. Compute and interpret the 95% CI(σ^2), where σ^2 is the population variance. Why would this interval be useful? You can use the online SOCR resource to get the values of the proper statistics. [Hint you'll need to construct a non symmetric CI, see class notes online.]

93	85	99	67	79	91	89	95	93	84	87	90

Last modified on by dinov@stat.ucla.edu.

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