UCLA STAT 110B

Applied Statistics for Engineering and the Sciences

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http://www.stat.ucla.edu/~dinov/courses_students.html

REVIEW
Estimation (ch. 6, 7)
Hypothesis Testing (ch. 8)

- Two Important Aspects of Statistical Inference
- Point Estimation – Estimate an unknown parameter, say θ, by some statistic computed from the given data which is referred to as a point estimator. Example: S^2 is a point estimate of σ^2
- Interval Estimation – A parameter is estimated by an interval that we are "reasonably sure" contains the true parameter value. Example: A 95% confidence interval for θ
- Hypothesis Testing – Test the validity of a hypothesis that we have in mind about a particular parameter using sample data.

Confidence Intervals for the Mean, μ

- Normally Distributed Population –
  - If σ known – construct with normal distribution
  - If σ unknown and n < 30 – construct with student's T distribution
- Arbitrarily Distributed Population -
  - If n >> 30 – apply Central Limit Theorem and use normal distribution

Confidence Interval for μ from a Normally Distributed Population, <u>σ known</u>

$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

Find the value $z_{\alpha/2}$ such that:

$$P\left(-z_{\alpha/2} < \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} < z_{\alpha/2} \right) = 1 - \alpha$$

Example

Construct a <u>**90%**</u> confidence interval for the mean of a normally distributed population specified by

$$\overline{x} = 5, \sigma^2 = 4, n = 25$$

Example cont.

Construct a **99%** confidence interval for the mean of a normally distributed population specified by

$$\overline{x} = 5, \sigma^2 = 4, n = 25$$

Example cont.

Construct a 99% confidence interval for the mean of a normally distributed population specified by

$$\overline{x} = 5, \sigma^2 = 4, n = 100$$

Confidence Interval for μ from a Normally Distributed Population, σ unknown

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$$

Find the value $t_{\alpha/2,n-1}$ such that:

$$P\left(-t_{\alpha/2,n-1} < \frac{\overline{X} - \mu}{S/\sqrt{n}} < t_{\alpha/2,n-1}\right) = 1 - \alpha$$

Example

Construct a 90% confidence interval for the mean of a normally distributed population specified by

$$\overline{x} = 5, S^2 = 4, n = 10$$

Large Sample (n>>30) Confidence Interval for μ from an Arbitrarily Distributed Population

Apply Central Limit Theorem

• Since n is large, the T-distribution limits to the standard normal. Hence, use a standard normal when computing confidence intervals regardless of whether σ is known or unknown.

$$T_{df=n-1} \sim T_o = \frac{\overline{X} - \mu}{S/\sqrt{n}} \approx Z_o = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

Example cont.

Construct a 90% confidence interval for the mean of an arbitrarily distributed population specified by

$$\overline{x} = 5, S^2 = 4, n = 35$$

Parameter (Point) Estimation

- (6.2) Two Ways of Proposing Point Estimators
- Method of Moments (MOMs):
  - Set your k parameters equal to your first k moments.
  - Solve. (e.g., Binomial, Exponential and Normal)
- Method of Maximum Likelihood (MLEs):
- 1. Write out likelihood for sample of size n.
- 2. Take natural log of the likelihood.
- 3. Take partial derivatives with respect to your k parameters.
- 4. Take second derivatives to check that a maximum exists.
- 5. Set 1st derivatives equal to zero and solve for MLEs. e.g., Binomial, Exponential and Normal

Parameter (Point) Estimation

- Suppose we flip a coin n=8 times and observe {T,H,T,H,H,T,H,H}. Estimate the value p = P(H).
- **Method of Moments Estimate p^:**
  - Set your k parameters equal to your first k moments.
- Let X = {# T's} ➔ np=8p=E(X)= Sample#H's = 5 ➔ p^=5/8.
- **Method of Maximum Likelihood Estimate p^:**
- 1. $f(x \mid p) = \binom{8}{5} p^5 (1-p)^3$ likelihood function.
- 2. $\ln\left(\binom{8}{5} p^5 (1-p)^3\right) = \ln\left(\binom{8}{5}\right) + 5 \times \ln(p) + 3 \times \ln(1-p)$
- 3. $\frac{d\left(\ln\left(\binom{8}{5}\right) + 5 \times \ln(p) + 3 \times \ln(1-p)\right)}{d\,p} = \frac{5}{p} - \frac{3}{1-p} = 0$

  $5(1-p) - 3p = 0 \Rightarrow p = 5/8$

Hypothesis Testing – the Likelihood Ratio Principle Example

- Let $\{X_1, \ldots, X_n\}=\{0.5, 0.3, 0.6, 0.1, 0.2\}$, weights, be IID $N(\mu, 1)$ ➔ $f(x;\mu)$. **Joint density** is $f(x_1,\ldots,x_n; \mu)=f(x_1;\mu)\times\ldots\times f(x_n;\mu)$.
- The likelihood function $L(p) = f(X_1,\ldots,X_n; p)$

$$L(\mu) = \lambda(x_1,\ldots,x_n) =$$

$$= e^{-\frac{(0.5-\mu)^2+(0.3-\mu)^2+(0.6-\mu)^2+(0.1-\mu)^2+(0.2-\mu)^2}{2}}$$

$$\ln(L) = (0.5-\mu)^2 + (0.3-\mu)^2 + (0.6-\mu)^2 + (0.1-\mu)^2 + (0.2-\mu)^2$$

$$0 = \frac{d\ln(L)}{d\mu} = -2(0.5-\mu) - 2(0.3-\mu) - 2(0.6-\mu) - 2(0.1-\mu) - 2(0.2-\mu) =$$

$$= +10\mu - 3.4 \Rightarrow \mu = 0.34$$

Hypothesis Testing

- In any problem there are two hypotheses:
- Null Hypothesis, H_o
- Alternative Hypothesis, H_a
- We want to gain inference about H_a, that is we want to establish this as being true.
- Our test results in one of two outcomes:
  - Reject H_o – implies that there is good reason to believe H_a true
  - Fail to reject H_o – implies that the data does not support that H_a is true; does not imply, however, that H_o is true

Hypothesis Testing - Motivation

- Point Estimates don't mean a thing unless you know how reliable the measurement is. Reporting an interval estimate at a certain level of confidence is a simple way to express uncertainty in your estimates.
- Hypothesis testing is about making one of two conclusions, reject or fail to reject, about a specified hypothesis, while knowing something about the probabilities of the two types of errors in your conclusion. The type I error you control by choosing α and your rejection region. If the sample size is fixed, the probability of a type II error can be found assuming a certain alternative is true. If the sample size hasn't been determined, you can find a sample size sufficient to ensure the probability of a type II error is below a desired level for a certain alternative.

Hypothesis Testing - Motivation

- Hypothesis Testing Steps in General:
- 1. Identify parameter of interest. Describe it in context.
- 2. Determine Null Value and State Null Hypothesis.
- 3. Determine alternative value/region and state null hypothesis.
- 4. Write Test Statistic without entering sample quantities.
- 5. State α and rejection region.
- 6. Calculate Test Statistic using necessary sample quantities.
- 7. State conclusion (reject or fail to reject) and interpret in context.

Hypothesis Testing - Motivation

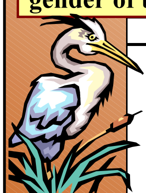
- What Test Do I Use when … ?
- 1. $X \sim N(\mu_{unknown}, \sigma^2_{known}) \rightarrow$ one-sample Z
- 2. $X \sim N(\mu_{unknown}, \sigma^2_{unknown}) \rightarrow$ one-sample T
- 3. $X \sim D(\mu_{unknown}, \sigma^2_{unknown})$, where D is fairly symmetric and n is moderately big $\rightarrow$ one-sample T
- 4. $X \sim D(\mu_{unknown}, \sigma^2_{unknown})$, where D is not symmetric and n is really big $\rightarrow$ one-sample T
- 5. $X \sim D(\mu_{unknown}, \sigma^2_{unknown})$, where D is not symmetric and n is not big $\rightarrow$ non-parametric, e.g. sign test.
- 6. $X \sim Bin(n_{known}, p_{unknown}) \rightarrow$ Z test for proportions

Hypothesis Testing - Motivation

- The above situations each have their corresponding power calculations and confidence intervals. In cases 1-4, the confidence intervals can be used to answer the hypothesis testing question. However, in case 6 the confidence intervals should not be used to answer the hypothesis testing question.

Hypothesis Testing – Statistical vs. Practical Significance

Is a second child gender influenced by the gender of the first child, in families with >1 kid?



First and Second Births by Sex

		Second Child	
		Male	Female
First Child	Male	3,202	2,776
	Female	2,620	2,792
	Total	5,822	5,568

- Research hypothesis needs to be formulated first before collecting/looking/interpreting the data that will be used to address it. Mothers whose 1st child is a girl are more likely to have a girl, as a second child, compared to mothers with boys as 1st children.
- Data: 20 yrs of birth records of 1 Hospital in Auckland, NZ.

Analysis of the birth-gender data – data summary

	Second Child	
Group	Number of births	Number of girls
1 (Previous child was girl)	5412	2792 (approx. 51.6%)
2 (Previous child was boy)	5978	2776 (approx. 46.4%)

- Let p_1=true proportion of girls in mothers with girl as first child, p_2=true proportion of girls in mothers with boy as first child. Parameter of interest is $p_1 - p_2$.
- H_0: $p_1 - p_2 = 0$ (skeptical reaction). H_a: $p_1 - p_2 > 0$ (research hypothesis)

Hypothesis testing as decision making

Decision Making

	Actual situation	
Decision made	H_0 is true	H_0 is false
Accept H_0 as true	OK	Type II error
Reject H_0 as false	Type I error	OK

- Sample sizes: $n_1=5412$, $n_2=5978$, Sample proportions (estimates) $\hat p_1 = 2792/5412 \approx 0.5159, \hat p_2 = 2776/5978 \approx 0.4644,$
- H_0: $p_1 - p_2 = 0$ (skeptical reaction). H_a: $p_1 - p_2 > 0$ (research hypothesis)

Analysis of the birth-gender data

- Samples are large enough to use Normal-approx. Since the two proportions come from totally diff. mothers they are independent → use formula 8.5.5.a

$$t_0 = \frac{\text{Estimate - HypothesizedValue}}{SE} = 5.49986 =$$

$$\frac{\hat p_1 - \hat p_2 - 0}{SE\left(\hat p_1 - \hat p_2\right)} = \frac{\hat p_1 - \hat p_2}{\sqrt{\frac{\hat p_1(1-\hat p_1)}{n_1} + \frac{\hat p_2(1-\hat p_2)}{n_2}}} =$$

$$P - value = \Pr(T \ge t_0) = 1.9 \times 10^{-8}$$

Analysis of the birth-gender data

- We have strong evidence to reject the H_0, and hence conclude mothers with first child a girl a more likely to have a girl as a second child.
- How much more likely? A 95% CI:

CI $(p_1 - p_2)$ =[0.033; 0.070]. And computed by:

$$\text{estimate} \pm z \times \text{SE} = \hat{p}_1 - \hat{p}_2 \pm 1.96 \times SE\left(\hat{p}_1 - \hat{p}_2\right) =$$

$$\hat{p}_1 - \hat{p}_2 \pm 1.96 \times \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} =$$

$$0.0515 \pm 1.96 \times 0.0093677 = [3\%; 7\%]$$

Hypothesis Testing – the Likelihood Ratio Principle

- Let $\{X_1, \ldots, X_n\}$ be a random sample from a density $f(x; p)$, where p is some population parameter. Then the **joint density** is $f(x_1,\ldots, x_n; p) = f(x_1; p) \times \ldots \times f(x_n; p)$.
- The likelihood function $L(p) = f(X_1,\ldots, X_n; p)$
- Testing: H_o: p is in Ω vs H_a: p is in Ω_a, where $\Omega \cap \Omega_a = 0$
  - Find max of L(p) in Ω.
  - Find max of L(p) in Ω_a.
  - Find likelihood ratio
  - Reject H_o if likelihood-ratio statistics λ is small (λ<k)

$$\lambda(x_1,\ldots,x_n) = \frac{\max_{p\in\Omega} L(p)}{\max_{p\in\Omega_a} L(p)}$$

Hypothesis Testing – the Likelihood Ratio Principle Example

- Let $\{X_1, \ldots, X_n\}$={0.5, -0.3, -0.6, 0.1, 0.2} be IID N(μ, 1) ➔ f(x;μ). The **joint density** is $f(x_1,\ldots,x_n; \mu)=f(x_1;\mu)\times\ldots\times f(x_n;\mu)$.
- The likelihood function $L(p) = f(X_1,\ldots,X_n; p)$
- Testing: H_o: σ>0.9 is in Ω vs H_a: σ<=0.9
- Reject H_o if likelihood-ratio statistics λ is small (λ<k)

$$\lambda_0 = \lambda(x_1,\ldots,x_n) = \frac{\max_{p\in\Omega} L(p)}{\max_{p\in\Omega_a} L(p)} =$$

ln(numer) = quadratic in μ!
ln(deno) = quadratic in μ!
Maximize both ➔ find ratio

$$\frac{\max_{\mu>0}\left(e^{-\frac{(0.5-\mu)^2+(-0.3-\mu)^2+(-0.6-\mu)^2+(0.1-\mu)^2+(0.2-\mu)^2}{2}}\right)}{\max_{\mu\le 0}\left(e^{-\frac{(0.5-\mu)^2+(-0.3-\mu)^2+(-0.6-\mu)^2+(0.1-\mu)^2+(0.2-\mu)^2}{2}}\right)}$$

Let P(Type I) = α

$t_o=1/\lambda_o \sim t_{\alpha,df=4}$

➔ one-sample T-test

Test Procedure

1. Calculate a Test Statistic (Example: z_o)
2. Specify a Rejection Region (Example: $|z_o| > z_{\alpha/2}$)
3. The null hypothesis is rejected iff the computed value for the statistic falls in the rejection region

Type I and Type II Errors

$$\alpha = \Pr\{\text{Reject } H_0 | H_0 \text{ is true}\}$$

$$\beta = \Pr\{\text{Fail to Reject } H_0 | H_0 \text{ is False}\}$$

- The value of α is specified by the experimenter
- The value of β is a function of **α, n, and δ** (the difference between the null hypothesized mean and the true mean). For a two sided hypothesis test of a normally distributed population

$$\beta = \Phi\left(Z_{\frac{\alpha}{2}} + \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-Z_{\frac{\alpha}{2}} + \frac{\delta\sqrt{n}}{\sigma}\right)$$

- It is not true that α =1- β (RHS=this is the test power!)

Type I and Type II Errors

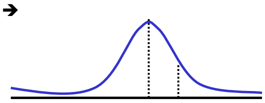
- Let μ denote the tread life of a certain type of tire. We would like to test whether the advertised tread life of 50,000 miles is accurate based on a sample of n=25 tires from a normally distributed population with σ = 1500. What are the null and alternative hypothesis? From these samples, we computed

$$\bar{x} = 49000$$

- Perform this hypothesis test at a level of significance α=0.05 (type I error). What is our conclusion? Now, assume that the true mean is actually 49,250 miles. Given samples of size n=25, what is the probability of a type II error, i.e. Pr(fail to reject the null hypothesis given that it is false)?

Type I and Type II Errors

- H_o: μ= 50,000 miles, sample-size n=25, Normal distribution with σ = 1500. H_1: μ < 50,000.
- α=0.05 (type I error). Assume true μ =49,250 miles. What is the P(type II error)? $\bar{X} \sim N(50{,}000;\ 300^2)$
  - α=0.05 ➔ $Z=(\bar{X} - \mu)/\sigma = -1.64$ ➔ $\bar{X} = Z*\sigma + \mu = 49{,}508$
  - β = P(fail to reject H_o | given that H_o is false) = ?
  - There is a **different β for each different true mean μ** .
  - $\beta = P(\bar{X}$ = avg.miles > μ = 49,508 $|\bar{X} \sim N(49{,}250;\ 1{,}500^2/25)$)=
  - Z=(49,508 - 49,250)/300 = 0.86 ➔
  - β = P(Z > 0.86) = 0.1939
  - Power of Test = 1- β = 0.8061



Another Example –Type I and Type II Errors

- One type of car is know to sustain no visible damage in 25% of 10-mph crash tests.A new bumper is proposed that increases this proportion. Let p be the new proportion of cars with no damage using the new bumpers. H_o: p=0.25, H_1: p>0.25.
- X = number of crushes/test with no damage in n=20 experiments. Under H_o we expect to get about n*p=5 no damage tests. Suppose we'd invest in new bumper technology if we get > 8 no damage tests ➔ rejection region R={8,9,…20}.
- Find α and β. How powerful is this test?

Another Example –Type I and Type II Errors

- H_o: p=0.25, H_1: p>0.25. X = number of crushes/test with no damage in n=20 experiments.
- X~Binomial(20, 0.25). Rejection region R={8,9,…20}.
- Find α =P(Type I) = P(X>=8 when X~Binomial(20, 0.25)).
- Use SOCR resource ➔ α =1-0.898 = 0.102
- Find β(p=0.3) =P (Type II) =
  - P(can't reject H_o | X~Binomial(20, 0.3))=P(X<7 | X~Binomial(20,0.3))
  - Use SOCR resource ➔ β =0.772
- Find β(p=0.5) =P (Type II) =
  - P(can't reject H_o | X~Binomial(20, 0.5))=P(X<7 | X~Binomial(20,0.5))
  - Use SOCR resource ➔ β =0.132

Comparing Population Means

- Generalize confidence intervals and tests for a single population parameter to that of two population parameters
- Consider two populations with means μ_1 and μ_2. We want to estimate $\mu_1 - \mu_2$, or possibly test H_o: $\mu_1 = \mu_2$
- Examples:
  - μ_1 = average crop yield using fertilizer 1
    μ_2 = average crop yield using fertilizer 2
  - μ_1 = women's average height
    μ_2 = men's average height

Assumptions

- $X_1, X_2, \ldots, X_m$ is a random sample from a population with mean μ_1 and variance σ_1^2
- $Y_1, Y_2, \ldots, Y_n$ is a random sample from a population with mean μ_2 and variance σ_2^2
- The X and Y samples are independent of one another

We will investigate using $\overline{X} - \overline{Y}$

as an estimator of the difference in the means $\mu_1 - \mu_2$

Expectation and Variance of $\overline{X} - \overline{Y}$

Test Procedures and Confidence Intervals for Normal Populations with Known Variances (9.1)

If both samples have a normal distribution, then the test statistic $\overline{X} - \overline{Y}$ has a normal distribution as well. It may be standardized by

Test Procedures and Confidence Intervals for Large Samples (9.1)

When both samples are large (n>30 and m>30):

- The CLT guarantees that regardless of the distribution of the data, $\overline{X}$ and $\overline{Y}$ will have a Normal distribution.
- The estimated standard deviations will be close to the population standard deviations

Example

Use the following data to construct a 95% confidence interval for $\mu_1 - \mu_2$.

$$m = 45, \overline{x} = 42500, s_1 = 2200, n = 45, \overline{y} = 40400, s_2 = 1900$$

Example

Consider the following data regarding boredom. The following table records the sample mean and standard deviation of the Boredom Proneness Rating for 97 male and 148 female college students surveyed. Test the hypothesis that the mean rating is higher for men than women at a .05 level of significance.

Gender	N	Avg	SamplSD
Male	97	10.4	4.83
Female	148	9.26	4.68

Two Sample t Test and Confidence Interval (9.2)

- The population variances are unknown
- At least one of the samples has a small sample size
- Assume each population is normally distributed. Experimentally, this may be established through normal probability plots.
- $\overline{X}$ and $\overline{Y}$ are standardized and distributed according to a t distribution

Example

Consider the following stress limits for different types of woods. Test the hypothesis that the true average stress limit for red oak exceeds that of Douglas fir by 1MPa

Type	N	Avg	SamplSD
Red Oak	14	8.48	0.79
Douglas Fir	10	6.65	1.28

Pooled t Procedures

- Not covered by the current edition of the book
- Assumes that the population variances are equal, i.e. $\sigma_1^2=\sigma_2^2$
- Outperforms the two sample t-test in β for a given level of α if the hypothesized equality of variances is true. Same is true for the confidence intervals
- May give erroneous results, however, if the variances are not equal, i.e. not robust to violations of this assumption

Comparing two means for independent samples

Suppose we have 2 samples/means/distributions as follows: $\{\bar{x}_1, N(\mu_1, \sigma_1)\}$ and $\{\bar{x}_2, N(\mu_2, \sigma_2)\}$. We've seen before that to make inference about $\mu_1 - \mu_2$ we can use a T-test for H_0: $\mu_1 - \mu_2 = 0$ with $t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{SE(\bar{x}_1 - \bar{x}_2)}$

And $CI(\mu_1 - \mu_2) = \bar{x}_1 - \bar{x}_2 \pm t \times SE(\bar{x}_1 - \bar{x}_2)$

If the 2 samples are independent we use the SE formula

$SE = \sqrt{s_1^2/n_1 + s_2^2/n_2}$ with $df = Min(n_1 - 1; n_2 - 1)$

This gives a conservative approach for hand calculation of an approximation to the what is known as the Welch procedure, which has a complicated exact formula.

Means for independent samples – equal or unequal variances?

Pooled T-test is used for samples with assumed equal variances. Under data Normal assumptions and equal variances of $(\bar{x}_1 - \bar{x}_2 - 0)/SE(\bar{x}_1 - \bar{x}_2)$, where

$$SE = s_p\sqrt{1/n_1 + 1/n_2};\ s_p^2 = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

is exactly Student's t distributed with $df = (n_1 + n_2 - 2)$

Here s_p is called the pooled estimate of the variance, since it pools info from the 2 samples to form a combined estimate of the single variance $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

The book recommends routine use of the Welch unequal variance method.

Analysis of Paired Data (9.3)

Paired Data - One set of individuals or objects; two observations made on each individual.

Unpaired Data -Two independent sets of individuals or objects; one observation per individual

Example

Consider testing whether a new drug significantly lowers blood pressure using 20 randomly selected patients

Unpaired Data – Randomly select 10 patients for the drug (1) and 10 for the placebo (2). Observe the magnitude of the reduction in blood pressure after taking medication. Test H_o: $\mu_1 = \mu_2$ vs. H_a: $\mu_1 > \mu_2$ using two-sample t-test

What about the age of the persons selected? Younger people may be more susceptible to a decrease in blood pressure than are older people. Can use pairing to "block" out age effect.

The Paired t-test

Assume that the data consists of n independently selected pairs $(X_1,Y_1), (X_2,Y_2),\ldots,(X_n,Y_n)$

Define $D_1=X_1-Y_1$, $D_2=X_2-Y_2$,…, $D_n=X_n-Y_n$. The D_i's are the differences within pairs. Check that the D_i's are normally distributed using a normal probability plot.

Let $D=\overline{X}-\overline{Y}$

Then $\mu_D =$

Hence testing H_o: $\mu_D=\Delta$

is equivalent to testing H_o: $\mu_1- \mu_2 =\Delta$

Since the D_i's are independent and normally distributed R.V'.s, we can use a one sample t-test to test the above hypothesis

Let $\overline{d}$ and s_D be the sample mean and sample standard deviation. It follows that the Confidence Interval and Hypothesis Test for the paired t-test are

Paired t- vs. Two-Sample t-test

The paired t-test has fewer degrees of freedom than the two-sample t-test. Hence, the two-sample t-test has a smaller β error for a fixed level of α than does the paired t-test. However, if there is a positive correlation between experimental units, the paired t-test will reduce the variance accordingly resulting in a more significant T statistic, where the two-sample t-test does not.

Paired t- vs. Two-Sample t-test

Use paired t-test if:

• There is great heterogeneity between experimental units and a large correlation within pairs

Use Two-Sample t-test if:

• The experimental units are relatively homogenous and the correlation between pairs is small

Inferences Concerning the Difference in Population Proportions (9.4)

• Previous sections (9.1,2,3): We compared the difference in the means (μ_1 - μ_2) of two different populations

• This section (9.4): We compare the difference in the proportions ($p_1 - p_2$) of two different populations