STAT 110 A, Probability & Statistics for Engineers I

UCLA Statistics

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SOLUTION HOMEWORK 3

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- **(HW_3_1) [Sec. 3.1, #4]**

In my perusal of a zip code directory, I found no 00000, nor did I find any zip codes with four zeros, a fact which was not obvious. Thus possible X values are 2, 3, 4, 5 (and not 0 or 1). X = 5 for the outcome 15213, X = 4 for the outcome 44074, and X = 3 for 94322.

- **(HW_3_2) [Sec. 3.2, #12]**

(a) $p_2(x)$ is the acceptable probability function, because all the probabilities are between zero and one and if probabilities add up to 1, whereas $p_1(x)$ and $p_3(x)$ are not acceptable because the summation of probabilities add up to a number which is less than one and greater than one respectively.

(b) $P(2 \le \boldsymbol{X} \le 4) = P(X=2)+P(X=3)+P(X=4)=0.5$

$P(\boldsymbol{X} \le 2) = 1- P(X \ge 3)=0.6$

$P(\boldsymbol{X} \ne 0) = 1- P(X \ge 1)=0.6$

(c) $p(x) = c\,(5 - x)$, then $\sum p(x)=1$. Hence, $\sum c\,(5 - x) = 1$. This implies that

$c((5-0)+(5-1)+(5-2)+(5-3)+(5-4))=1$, that is $c(15)=1$ and so $c=1/15$.

- **(HW_3_3) [Sec. 3.2, #13]**

a. $P(X \le 3) = p(0) + p(1) + p(2) + p(3) = .10+.15+.20+.25 = .70$

b. $P(X < 3) = P(X \le 2) = p(0) + p(1) + p(2) = .45$

c. $P(3 \le X) = p(3) + p(4) + p(5) + p(6) = .55$

d. $P(2 \le X \le 5) = p(2) + p(3) + p(4) + p(5) = .71$

e. The number of lines not in use is $6 - X$, so $6 - X = 2$ is equivalent to $X = 4$, $6 - X = 3$ to $X = 3$, and $6 - X = 4$ to $X = 2$. Thus we desire $P(2 \le X \le 4) = p(2) + p(3) + p(4) = .65$

f. $6 - X \ge 4$ if $6 - 4 \ge X$, i.e. $2 \ge X$, or $X \le 2$, and $P(X \le 2) = .10+.15+.20 = .45$

- **(HW_3_4) [Sec. 3.3, #28]**

a. $E(X) = \sum_{x=0}^{4} x \cdot p(x) = (0)(.08) + (1)(.15) + (2)(.45) + (3)(.27) + (4)(.05) = 2.06$

b. $V(X) = \sum_{x=0}^{4} (x - 2.06)^2 \cdot p(x) = (0 - 2.06)^2(.08) + \ldots + (4 - 2.06)^2(.05)$

$= .339488+.168540+.001620+.238572+.188180 = .9364$

c. $\sigma_x = \sqrt{.9364} = .9677$

d. $V(X) = \left[\sum_{x=0}^{4} x^2 \cdot p(x)\right] - (2.06)^2 = 5.1800 - 4.2436 = .9364$

- **(HW_3_5) [Sec. 3.3, #31]**

a. $E(X) = (13.5)(.2) + (15.9)(.5) + (19.1)(.3) = 16.38$,
$E(X^2) = (13.5)^2(.2) + (15.9)^2(.5) + (19.1)^2(.3) = 272.298$,
$V(X) = 272.298 - (16.38)^2 = 3.9936$

b. $E(25X - 8.5) = 25\,E(X) - 8.5 = (25)(16.38) - 8.5 = 401$

c. $V(25X - 8.5) = V(25X) = (25)^2 V(X) = (625)(3.9936) = 2496$

d. $E[h(X)] = E[X - .01X^2] = E(X) - .01E(X^2) = 16.38 - 2.72 = 13.66$