STAT 110 A, Probability & Statistics for Engineers I

UCLA Statistics

http://www.stat.ucla.edu/~dinov/courses_students.html

SOLUTION HOMEWORK 4

(HW_4_1) [Sec. 3.5, #69]

a. With S = a female child and F = a male child, let X = the number of F's before the 2^{nd} S. Then P(X = x) = nb(x;2, .5)

b. P(exactly 4 children) = P(exactly 2 males)
= nb(2;2,.5) = (3)(.0625) = .188

c. P(at most 4 children) = P(X ≤ 2)
$= \sum_{x=0}^{2} nb(x;2,.5)$ = .25+2(.25)(.5) + 3(.0625) = .688

d. E(X) = $\frac{(2)(.5)}{.5} = 2$, so the expected number of children = E(X + 2)
= E(X) + 2 = 4

(HW_4_2) [Sec. 3.6, #76]

a. P(X = 1) = F(1;2) – F(0;2) = .982 - .819 = .163

b. P(X ≥ 2) = 1 – P(X ≤ 1) = 1 – F(1;2) = 1 - .982 = .018

c. P(1^{st} doesn't ∩ 2^{nd} doesn't) = P(1^{st} doesn't) · P(2^{nd} doesn't)
= (.819)(.819) = .671

(HW_4_3) [Sec. 4.1, #2]

F(x) = $\frac{1}{10}$ for –5 ≤ x ≤ 5, and = 0 otherwise

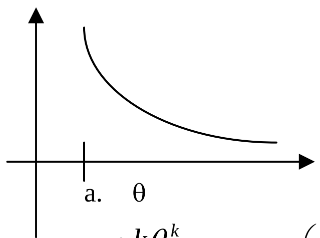
a. P(X < 0) = $\int_{-5}^{0} \frac{1}{10}\,dx = .5$

b. P(-2.5 < X < 2.5) = $\int_{-2.5}^{2.5} \frac{1}{10}\,dx = .5$

c. P(-2 ≤ X ≤ 3) = $\int_{-2}^{3} \frac{1}{10} dx = .5$

d. P(k < X < k + 4) = $\int_{k}^{k+4} \frac{1}{10} dx = \frac{x}{10}\Big]_{k}^{k+4} = \frac{1}{10}[(k+4)-k] = .4$

a)



b) $= \int_{-\infty}^{\infty} f(x;k,\theta)dx = \int_{\theta}^{\infty} \frac{k\theta^k}{x^{k+1}}dx = \theta^k \cdot \left(-\frac{1}{x^k}\right)\Bigg]_{\theta}^{\infty} = \frac{\theta^k}{\theta^k} = 1$

c) P(X ≤ b) = $\int_{\theta}^{b} \frac{k\theta^k}{x^{k+1}}dx = \theta^k \cdot \left(-\frac{1}{x^k}\right)\Bigg]_{\theta}^{b} = 1 - \left(\frac{\theta}{b}\right)^k$

d) P(a ≤ X ≤ b) = $\int_{a}^{b} \frac{k\theta^k}{x^{k+1}}dx = \theta^k \cdot \left(-\frac{1}{x^k}\right)\Bigg]_{a}^{b} = \left(\frac{\theta}{a}\right)^k - \left(\frac{\theta}{b}\right)^k$

e) E(X) = $\int_{\theta}^{\infty} x \cdot \frac{k\theta^k}{x^{k+1}} dx = k\theta^k \int_{\theta}^{\infty} \frac{1}{x^k} dx = \frac{k\theta^k x^{-k+1}}{-k+1}\Bigg]_{\theta}^{\infty} = \frac{k\theta}{k-1}$

f) E(X) = ∞

g) E(X^2) = $k\theta^k \int_{\theta}^{\infty} \frac{1}{x^{k-1}} dx = \frac{k\theta^2}{k-2}$, so

Var(X) = $\left(\frac{k\theta^2}{k-2}\right) - \left(\frac{k\theta}{k-1}\right)^2 = \frac{k\theta^2}{(k-2)(k-1)^2}$

h) Var(x) = ∞, since E(X^2) = ∞.

(HW_4_5) [Sec. 4.3, #33]

a. P(X ≥ 10) = P(Z ≥ .43) = 1 - Φ(.43) = 1 - .6664 = .3336.
P(X > 10) = P(X ≥ 10) = .3336, since for any continuous distribution, P(x = a) = 0.

b. $P(X > 20) = P(Z > 4) \approx 0$

c. $P(5 \le X \le 10) = P(-1.36 \le Z \le .43) = \Phi(.43) - \Phi(-1.36) = .6664 - .0869 = .5795$

d. $P(8.8 - c \le X \le 8.8 + c) = .98$, so $8.8 - c$ and $8.8 + c$ are at the 1st and the 99th percentile of the given distribution, respectively. The 1st percentile of the standard normal distribution has the value –2.33, so
$8.8 - c = \mu + (-2.33)\sigma = 8.8 - 2.33(2.8) \Rightarrow c = 2.33(2.8) = 6.524$.

(HW_4_6) [Sec. 4.3, #42]

a. $P(67 \le X \le 75) = P(-1.00 \le Z \le 1.67) = .7938$

b. $P(70 - c \le X \le 70 + c) = P\left(\frac{-c}{3} \le Z \le \frac{c}{3}\right) = 2\Phi(\frac{c}{3}) - 1 = .95 \Rightarrow \Phi(\frac{c}{3}) = .9750$

$$\frac{c}{3} = 1.96 \Rightarrow c = 5.88$$

c. 10·P(a single one is acceptable) = 7.935

d. $p = P(X < 73.84) = P(Z < 1.28) = .9$, so $P(Y \le 8) = B(8;10,.9) = .264$