

STAT 110 A, Probability & Statistics for Engineers I

UCLA Statistics

http://www.stat.ucla.edu/~dinov/courses_students.html

SOLUTION HOMEWORK 5

(There is a total of 100 points for this assignment.)

Problem 1 (Total: 16 points; 4 points each)

a) Yes.

b) $\{X \geq t\} = A1 \cap A2 \cap A3 \cap A4 \cap A5$

c) $P(X \geq t) = P(A1)P(A2)P(A3)P(A4)P(A5) = (e^{-\lambda t})^5 = e^{-0.05t}$

$F(t) = P(X \leq t) = 1 - e^{-0.05t}$

d) $f(t) = 0.05 e^{-0.05t}$, X has an exponential distribution with $\lambda = 0.05$.

Problem 2 (Total: 16 points; 4 points each)

X ~ normal distribution; $P = 0.1$; $n=200$; $np = 20$; $npq = 18$

a) $P(X \leq 30) = \Phi\left(\frac{30 + 0.5 - 20}{\sqrt{18}}\right) = 0.9932$

b) $P(X < 30) = P(X \leq 29) = \Phi(2.24) = 0.9875$

c) $P(15 \leq X \leq 25) = P(X \leq 25) - P(X \leq 14) = \Phi(1.30) - \Phi(-1.30) = 0.8064$

d) $X = 20 - 0.44(\sqrt{18}) = 18.1332$

Problem 3 (Total: 15 points; 5 points each)

a) $f(y|x) = \frac{f(x, y)}{f_x(x)} = \frac{\frac{9}{26} y^2 (x+1)^2}{\int_0^1 \frac{9}{26} y^2 (x+1)^2 dy} = 3y^2$

b) $P(Y < 1/2 | X < 1/2) = \frac{P(0 < x < 1/2; 0 < y < 1/2)}{P(0 < x < 1/2)} = \frac{\int_0^{1/2} \int_0^{1/2} f(x, y) dy dx}{\int_0^{1/2} f_x(x) dx} = \frac{1}{8}$

c) $E(Y | X = x) = \int y f(y | x) dy = \int_0^1 y (3y^2) dy = \frac{3}{4}$

Problem4 (Total: 16 points; 4 points each)

Refer to your lecture notes.

Problem5 (Total: 14 points; 7 points each)

$$a) E(X) = \int_0^1 x(\theta+1)x^\theta dx = \frac{\theta+1}{\theta+2} = 1 - \frac{1}{\theta+2} \quad \text{So } \bar{X} = 1 - \frac{1}{\theta+2} \Rightarrow \hat{\theta} = \frac{1}{1-\bar{X}} - 2$$

Since $\bar{X} = 0.80$, $\hat{\theta} = 3$.

$$b) f(x_1, \dots, x_n; \theta) = (\theta+1)^n (x_1 x_2 \cdots x_n)^\theta \quad \text{so } \log f = n \ln(\theta+1) + \theta \sum \ln(x_i)$$

$$\Rightarrow \hat{\theta} = -\frac{n}{\sum \ln(x_i)} - 1 \quad \text{so } \hat{\theta} = 3.12$$

Problem 6 (Total: 10 points)

$$95\% \text{ C.I. is } 1.95 \pm 2.365(0.22/\sqrt{8}) = (1.766, 2.134)$$

Problem 7 (Total: 13 points)

$$p = 260/500 = 0.52$$

$$\text{s.d.} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.52)(0.48)}{500}} = 0.02234 \quad (1 \text{ point})$$

$$95\% \text{ C.I. is } 0.52 \pm (1.96)(0.02234) = (0.476, 0.564) \quad (6 \text{ points})$$

$$0.02 = \sqrt{\frac{(0.52)(0.48)}{n}} \quad \text{so } n = 624 \quad (6 \text{ points})$$